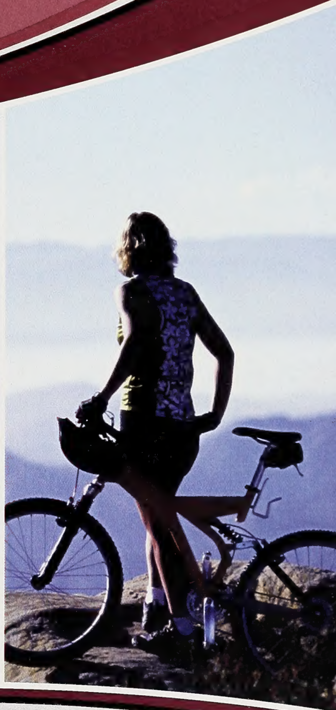
3

MATHEMATICS 24



Module 3
APPLYING GEOMETRY

Learning Technologies Branch

Alberta LEARNING

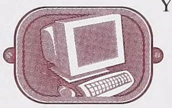
MATHEMATICS 24

Module 3
APPLYING GEOMETRY

Mathematics 24
Module 3: Applying Geometry
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2589-6

The Learning Technologies Branch acknowledges with appreciation the Alberta Distance Learning Centre and Pembina Hills Regional Division No. 7 for their review of this Student Module Booklet.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	



You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome to

MATHEMATICS 24

Module Three

Mathematics 24 contains six modules. You should work through the modules in order (from 1 to 6) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
INDEPENDENT LIVING

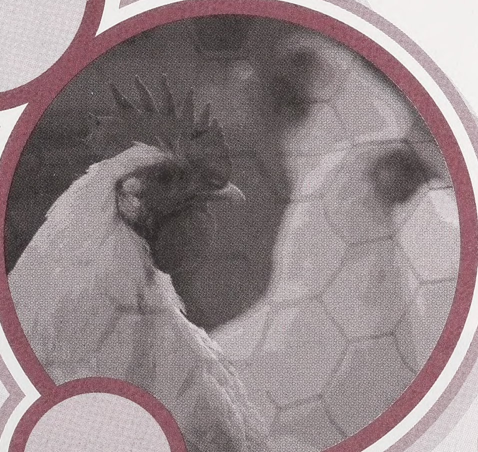
Module 2
WHEELS

Module 3
APPLYING GEOMETRY

Module 4
MAPS, DATA, and PROBABILITY

Module 5
STATISTICS

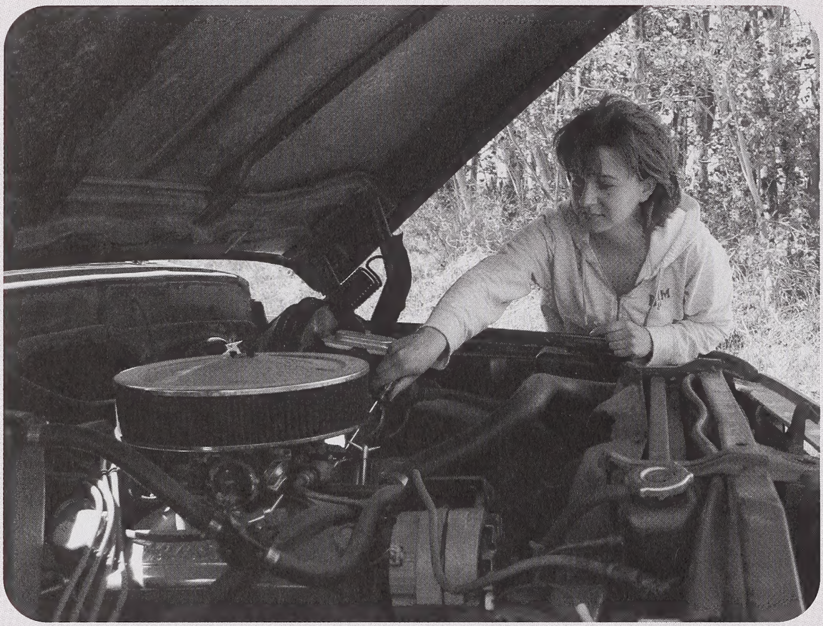
Module 6
DESIGN and CONSTRUCTION



Module 1 contains general information about the course components, required resources, visual cues, assessment and feedback, and strategies for completing your work. If you do not have access to Module 1, contact your teacher to obtain this important information.

CONTENTS

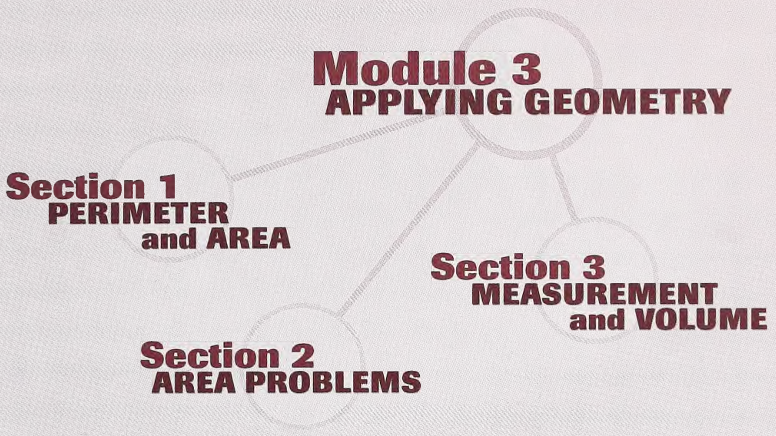
MODULE OVERVIEW



Shay and her brother Jack have been restoring their dad's truck for most of the winter. Now they finally get to enjoy the fruits of their hard work. When they'd pulled the engine and started working on it, it was a bit scary. Were those new rings the right size? They had measured the cylinders correctly, hadn't they? Their confidence had grown as the restoration progressed. Now they knew that all that measuring, planning, and labour had paid off.

Trucks and their engines are complicated pieces of engineering. It's hard enough to figure out how to fix one, let alone design one. How do the engineers get all the pieces together in the small space under the hood? It requires knowledge about the space needed and about putting things together without wasting any space. It's part experience and part knowledge, just like Shay and Jack's restoration work.

In this module you will learn how to find the perimeter, area, and volume of many objects. You will also see how to use these "sizes" to solve everyday problems.



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The suggested mark distribution is as follows. Be sure to check with your teacher if this mark allocation is valid for you. Some teachers like to include other reviews and assignments.

Assignment Booklet 3A	
Section 1 Assignment	27 marks
Section 2 Assignment	19 marks
Assignment Booklet 3B	
Section 3 Assignment	21 marks
Final Module Assignment	33 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 3A to your teacher before you begin Section 3. You will submit Assignment Booklet 3B to your teacher at the end of this module.

SECTION 1

Perimeter and Area

People have always enjoyed the sight of horses running. There is something wonderfully free about such a large animal running just for the fun of it. Of course, in their natural state, running was one of their defences against predators. It also proved useful for humans. Imagine how much faster you could pursue food on a horse compared to on foot.

It also proved to be a problem for humans. How can you use horses to help you if they can just get up and run away? To keep horses from roaming too far, people have fenced them in. They are free to run only inside their paddocks. They have some space and some freedom, but it has a finite boundary.

In this section you will learn about boundaries and their lengths (also known as perimeter). You will also learn about the space inside a shape, which is known as area.

LESSON 1

Using Perimeter to Solve Problems

In this lesson you will learn how to find the perimeter of different shapes.

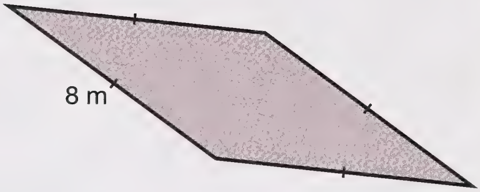
These runners are preparing for a race around an oval track. In order for the race to be fair, all the runners must run the same distance. However, because the track is oval, runners on the inside lanes have quite an advantage because the inner lanes are shorter. To make up the difference, the runners in the outer lanes are moved forward so that everyone runs the same distance.

Finding the distance around something is not just useful for races. **Perimeter** shows up in a lot of places, as you will see in this lesson.

Turn to page 139 in your textbook. Read the "get thinking" box at the top of the page. With these questions in mind, turn to page 140. Check out the "closer look" box and then work through Example 1 on page 140 to see the perimeters of different shapes being estimated and calculated.

Example

What is the perimeter of the **rhombus** shown to the right? Since all sides of a rhombus are the same length, there are four sides, each with a length of 8 m.



$$P = 8\text{ m} + 8\text{ m} + 8\text{ m} + 8\text{ m}$$
$$= 32\text{ m}$$

The perimeter of this rhombus is 32 m. (Did you notice the little marks on each side of the diagram? Sides marked with the same symbol or mark have the same length.)

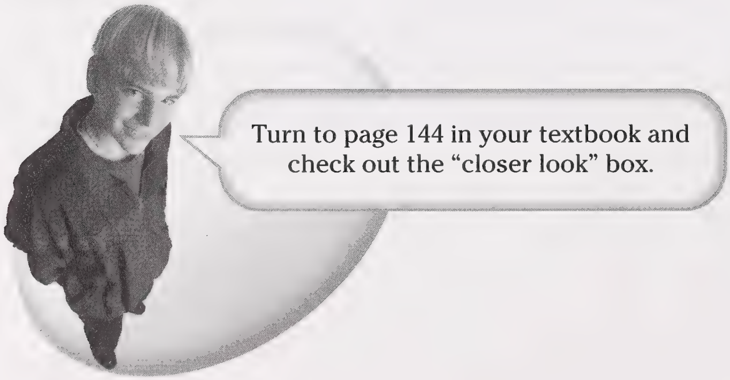
You've seen several examples with perimeters being found. Now you get to practise on some questions.

1. Turn to page 143 in your textbook. Complete questions 1.b., 1.c., 2.b., 2.c., and 2.e. of "Put into Practice."

Check your answers on pages 48 and 49 in the Appendix.

You've had some experience with the perimeter of straight-sided objects. Now it's time to find the perimeter of circular objects.

Turn to page 141 in your textbook and study Example 2. Notice that for circular objects, the perimeter has the special name **circumference**. You will also be reminded of a special number, π (**pi**), that is associated with **circles**.



Example

What is the perimeter of the shape shown to the right? Round your answer to the nearest millimetre.

The diameter of the circle is given as 55 mm. The perimeter of a circle is also known as the **circumference**.

$$\begin{aligned} C &= \pi d \\ &= \pi \times 55 \text{ mm} \\ &\doteq 172.787\,595\,9 \text{ mm} \end{aligned}$$

The perimeter of this shape is about 173 mm.

You've seen the examples; now it's time for you to solve some circumference problems.

2. Continue on page 144 in your textbook. Complete questions 3, 4.a., and 4.c. of "Put into Practice."

Check your answers on page 49 in the Appendix.

Sometimes, problems arise that relate to perimeter, but that are not exactly perimeter problems. Turn to page 142 in your textbook. Work through Example 3 carefully. Do you remember the five steps for solving problems?

Now it's time to solve some perimeter-based problems.

3. Turn to page 145 in your textbook. Complete questions 5 and 6 of "Put into Practice."

Check your answers on pages 50 and 51 in the Appendix.

In this lesson you have seen how to find the perimeter of shapes, like **squares rectangles**, and circles.

Turn to the Section 1 Assignment in Assignment Booklet 3A. Answer question 1.

LESSON 2

Areas of Rectangles, Triangles, and Parallelograms

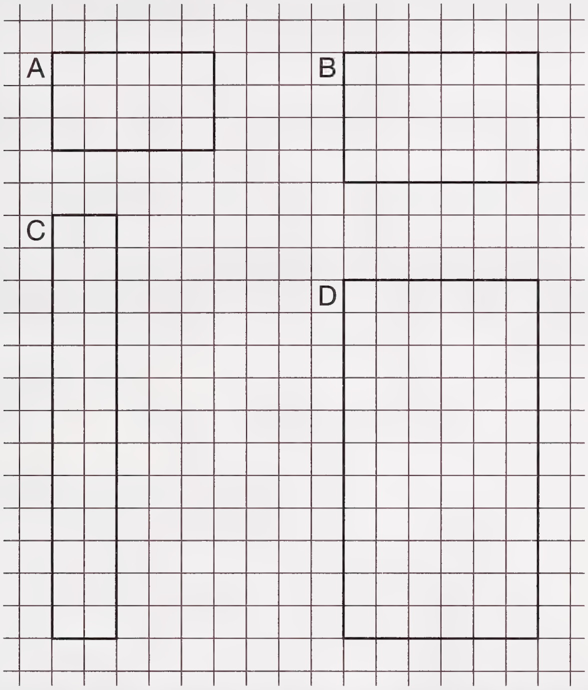
In this lesson you will find the areas of rectangles, triangles, and parallelograms.

The sight of cattle eating at a feedlot is quite common in Alberta. Having the right amount of land is quite important to the feedlot owner. While land in rural Alberta is not nearly as expensive as land in cities and towns, it still has a cost. Good farming and business practices call for using enough land and resources but not any extra.

Knowing how much space something takes up is not just used in farming. **Area** is an important concept in many aspects of daily living.

Turn to page 147 in your textbook. Read the "get thinking" box at the top of the page. Keep these questions in mind as you work through the following investigations.

1. Turn to pages 147 and 148 in your textbook and complete questions 2 to 6 of "Investigation 1: How can you determine the areas of rectangular shapes?" Use rectangles A, B, C, and D to answer the questions.



Check your answers on page 52 in the Appendix.

Example

What is the area of the rectangle?

The rectangle has a base of 5 cm and a height of 3 cm.

$$\begin{aligned} A &= bh \\ &= 5 \text{ cm} \times 3 \text{ cm} \\ &= 15 \text{ cm}^2 \end{aligned}$$

The area of the rectangle is 15 cm^2.

Rectangle
$A = bh$

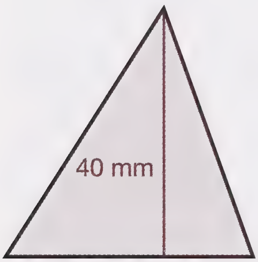
Rectangular areas are the easiest to find. **Triangles** are another simple shape. Their areas are a little trickier to find, but not much.

2. Turn to pages 148 and 149 in your textbook. Work through questions 1 to 3 of "Investigation 2: How can you determine the areas of triangles?"

Check your answers on pages 53 to 56 in the Appendix.

Example

What is the area of the following triangle?



The triangle has a base of 40 mm and a height of 40 mm.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 40\text{ mm} \times 40\text{ mm} \\ &= 800\text{ mm}^2 \end{aligned}$$

The area of the triangle is 800 mm^2.

3. Turn to page 152 in your textbook and answer questions 1.b., 1.c., 2.c., and 2.e. of "Put into Practice."

Check your answers on page 57 in the Appendix.

Triangle
$A = \frac{1}{2}bh$

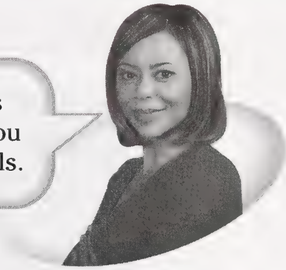
Triangular areas were pretty easy to find. Another shape with an area that is easy to find is the **parallelogram**.

4. Turn to page 150 in the textbook. Work through question 1 of “Investigation 3: Determining Areas of More Parallelograms.”

Check your answers on page 58 in the Appendix.

Parallelogram
$A = bh$

The exploration of finding areas of shapes is over for now. It’s time to review what you have explored and then practise your skills.



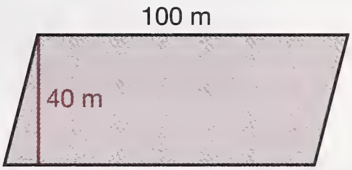
Turn to page 151 in the textbook. Read the “closer look” box at the top of the page. Then work through the example carefully.

Example

What is the area of the following shape?

The parallelogram has a base of 100 m and a height of 40 m.

$$\begin{aligned} A &= bh \\ &= 100 \text{ m} \times 40 \text{ m} \\ &= 4000 \text{ m}^2 \end{aligned}$$



The area of the parallelogram is 4000 m^2.

5. Turn to page 153 in the textbook and answer questions 2.b., 3.a., 3.b., and 3.d. of “Put into Practice.”

Check your answers on pages 59 and 60 in the Appendix.

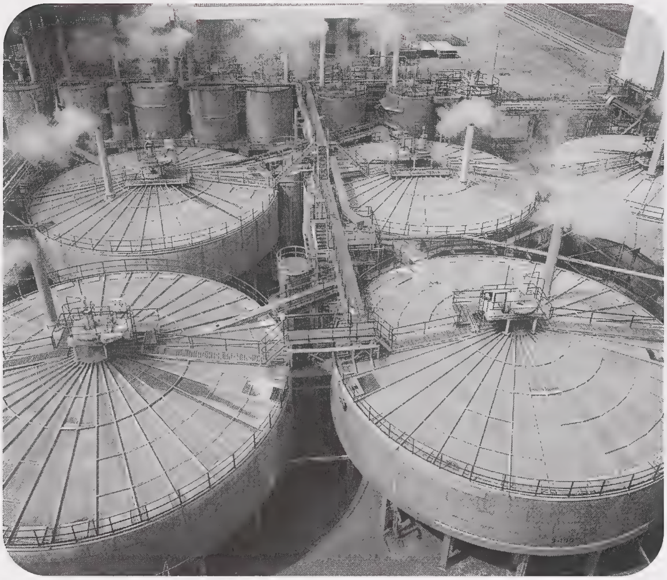
In this lesson you have seen how to find the areas of rectangles, triangles, and parallelograms.

Turn to the Section 1 Assignment in Assignment Booklet 3A. Answer question 2.

LESSON 3

Areas of Circles

In this lesson you will find the areas of circles.

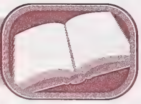


In an industrial setting, you would expect to find containers that are best for the job at hand. Are you surprised to find circular storage tanks in such a setting?

A storage tank in the shape of a cylinder has some very special features that make it attractive and useful in industry.

These same features make cylindrical containers useful in the home too. Just check out your cupboards and fridge. You'll find lots of things packed in cylinders. There will be shampoo bottles, cans of pop, cans of vegetables, and even pails of ice cream.

You'll have to wait until later in the module to learn what some of the advantages of cylindrical containers are. For now, you'll have to be satisfied with learning how to find the space a circular shape takes up.



Turn to page 154 in your textbook. Read the "get thinking" box at the top of the page. Then read the hint near the top of the page. Use this information to help you complete the investigation.

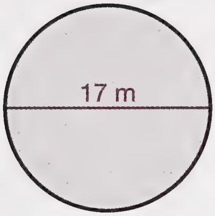
1. Complete questions 1 to 3 of "Investigation: How can you estimate the area of a circle?"

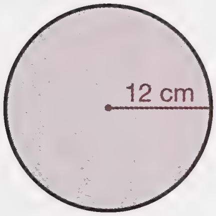
Check your answers on page 60 in the Appendix.

Turn to page 155 in the textbook. Read the “closer look” box at the top of the page for a review of how the parts of a circle relate to its area. Then work through Example 1.

Example

What are the areas of the following circles? Round your answers to one decimal place.





This circle has a **diameter** of 17 m. Its radius is $17 \div 2 = 8.5$ m.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (8.5 \text{ m})^2 \\ &\doteq 226.980\,069\,2 \text{ m}^2 \end{aligned}$$

The area of this circle is about 227.0 m^2.

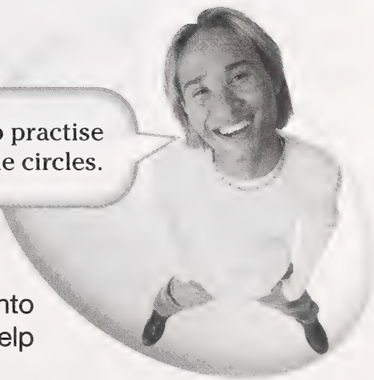
This circle has a **radius** of 12 cm.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (12 \text{ cm})^2 \\ &\doteq 452.389\,342\,1 \text{ cm}^2 \end{aligned}$$

The area of this circle is about 452.4 cm^2.

Circle
$A = \pi r^2$

Now it’s time to practise finding the area of some circles.



2. Turn to page 156 in your textbook and complete questions 1.b., 1.c., and 2 of “Put into Practice.” The “reminder” on the page may help you with question 2.

Check your answers on page 61 in the Appendix.

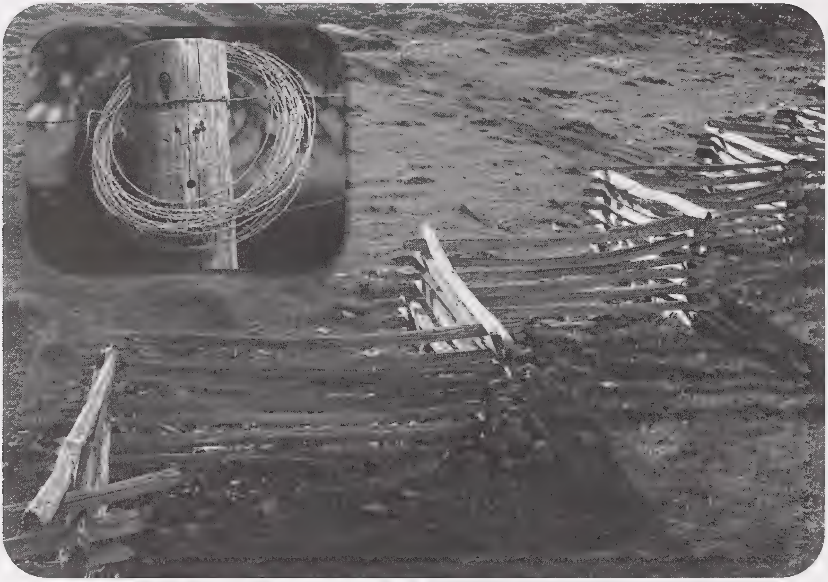
In this lesson you've seen how to find the areas of circles.

Turn to the Section 1 Assignment in Assignment Booklet 3A. Answer question 3.

CONCLUSION

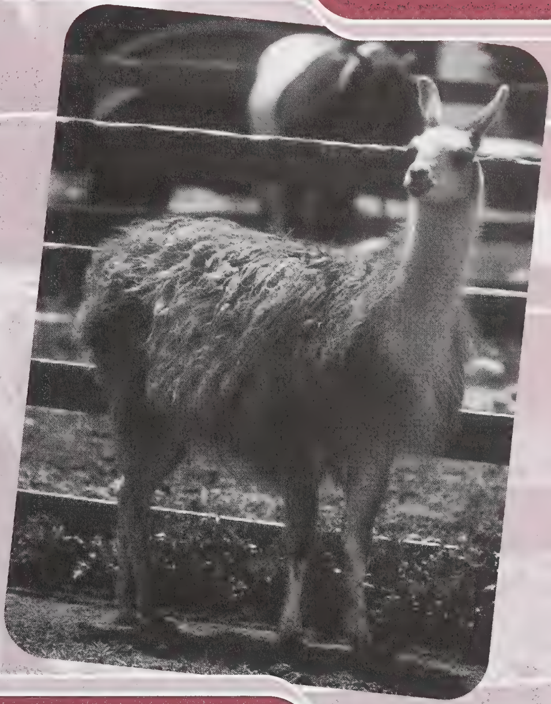
In this section you used formulas to find the areas and perimeters of rectangles, parallelograms, triangles, and circles.

As farm livestock gets more diverse, fencing needs change. While a simple rail fence or barbed-wire fence works for containing cattle or horses, higher and stronger fences are needed for bison, deer, or elk.



How would you go about finding the perimeter and area of a field with a fence like the one above?

SECTION 2



Area Problems

When you think of farm animals in Alberta, what's the first animal that comes to mind? Most people would say cows. Without a picture prompt, very few would mention llamas.

Do you know if llamas and cows get along, or do they have to be separated? Fences are used to separate animals on a farm. They enclose the space that each group of animals gets to use. Fencing can cost a lot of money, however, so enclosures usually aren't much bigger than needed. How would the proper size of pen be calculated?

In this section you will work on problems that involve area. You will study how to find the areas of different-shaped objects and how to use area in solving everyday problems. You will also see how a computer spreadsheet program can help with these problems.

LESSON 1

Solving Area Problems with 2-D Shapes

In this lesson you will solve area problems with two-dimensional shapes.

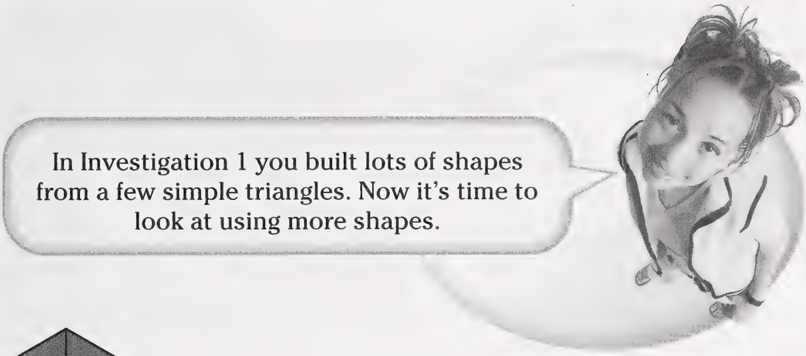
The dried mud in the photo has cracked into pieces that have many different shapes. None of the smaller shapes are very complicated, but they are not simple shapes, like circles and rectangles.

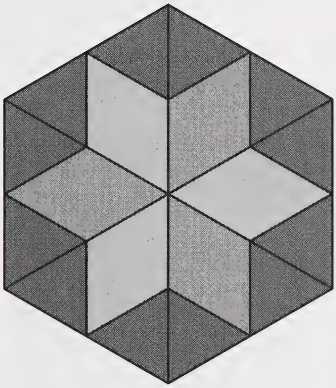
Section 1 dealt with the space taken up by simple shapes. Now it's time to study the space taken up by more complicated shapes.

Turn to page 157 in your textbook. Read the "get thinking" box at the top of the page.

1. Complete questions 1 and 2 of "Investigation 1: How can you determine the area of irregular or composite shapes?"

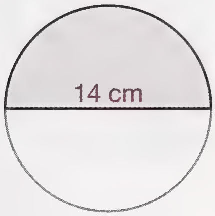
Check your answers on pages 62 to 64 in the Appendix.





Turn to page 158 in the textbook and read the "closer look" box. Keep these ideas in mind as you study Example 1. Then read the "reminder" at the top left of page 159.

Example



What is the area of the coloured part of the circle on the right? Round your answer to two decimal places.

The first step is to find the area of the whole circle.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{14\text{ cm}}{2}\right)^2 \\ &= \pi(7\text{ cm})^2 \\ &\doteq 153.938\,04\text{ cm}^2 \end{aligned}$$

Then, since the coloured part is half of the circle, divide the area of the circle by 2.

$$153.938\,04\text{ cm}^2 \div 2 \doteq 76.969\,020\,01\text{ cm}^2$$

The area of the coloured part of the circle is about 76.97 cm^2.

2. Turn to pages 159 to 163 in the textbook, and answer questions 1.a., 1.b., 2.a., 2.c., 2.e., 2.f., 3.a., 3.b., 3.c., 3.d., 3.f., 3.h., 4, and 5 of "Put into Practice."

Check your answers on pages 65 to 69 in the Appendix.

You can practise finding the area of irregular shapes on the multimedia CD. The segment "Making Complicated Area Problems Simpler" provides some sample shapes to simplify.

In this lesson you studied how to find the areas of irregular two-dimensional shapes.

Turn to the Section 2 Assignment in Assignment Booklet 3A. Answer question 1.

LESSON 2

Solving Area Problems with 3-D Solids

In this lesson you will solve area problems with three-dimensional shapes.

Buildings don't have to be boxlike. They can have interesting shapes like those shown below.

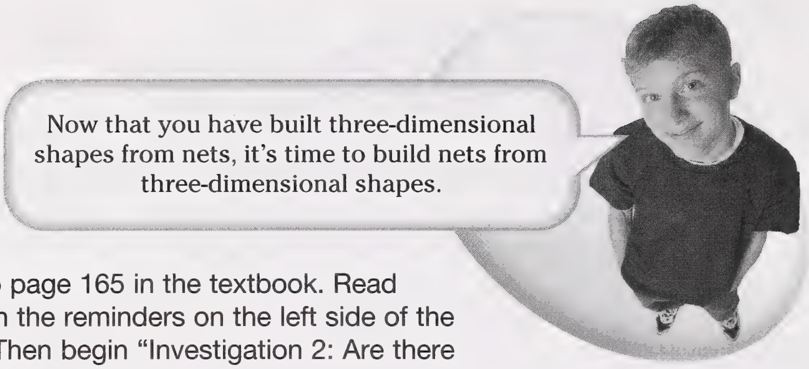
There won't be as much usable floor space in buildings like these as there would be in a boxlike building of the same height and with the same base, but these are much more interesting to look at.

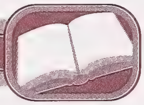
Why might a builder make a building that doesn't maximize the space inside?

Turn to page 164 in your textbook. Read the "get thinking" box at the top of the page.

1. Complete questions 1 to 7 of "Investigation 1: How can you determine the surface area of 3-dimensional solids?" Use the **nets** at the end of the Appendix for this investigation.

Check your answers on pages 69 to 71 in the Appendix.





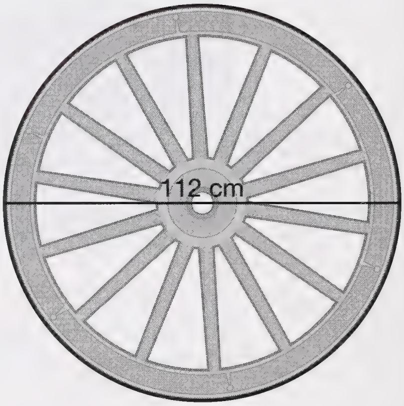
Turn to page 165 in the textbook. Read through the reminders on the left side of the page. Then begin "Investigation 2: Are there formulas for surface area?"

2. On page 165 in the textbook, complete questions 1 and 2 of "Investigation 2: Are there formulas for **surface area**?" (**Note:** You only need to use one cereal box or similar box for question 1. Toilet paper and paper towels usually have a **cylinder** in the centre of the roll that you can use for question 2.)

Check your answers on pages 72 and 73 in the Appendix.

Example

Stagecoaches and wagons had wheels made of wood. These wheels had a band of metal around the outside for strength and durability. A particular wheel was 112 cm in diameter. The metal band was 15 cm wide. How many square centimetres of metal were needed to make the band?



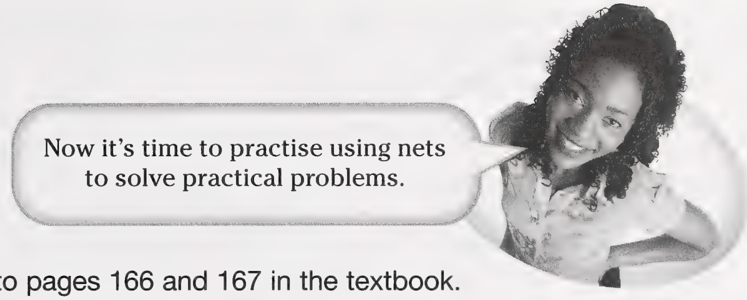
$$C = \pi d$$
$$= 112\pi \text{ cm}$$

The net for this metal band would look like the following. (Its length would have to be the same as the circumference of the wheel.)

The area of this band could be found as follows:

$$A = bh$$
$$= 112\pi \text{ cm} \times 15 \text{ cm}$$
$$\doteq 5277.875\ 658 \text{ cm}^2$$

The area of this band would be about 5278 cm^2.



3. Turn to pages 166 and 167 in the textbook.
Complete questions 1 to 4 of "Put into Practice."

Check your answers on pages 73 to 76 in the Appendix.

In this lesson you found the surface area of three-dimensional shapes and saw how making a net for a solid can help you find the solid's surface area.

the Section 2 Assignment in Assignment Booklet 3A.
Answer question 2.

LESSON 3

Are Perimeter and Area Related?

In this lesson you will investigate how perimeter and area are related.

The houses in the photo appear to be identical (except for their colours and decorations on the outside).

How might these houses have different sizes even though they look so alike?

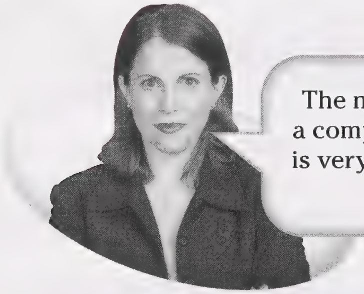
Turn to page 168 in the textbook. Read the “get thinking” box at the top of the page. Then continue by working through “Investigation 1: What happens to area when you change the dimensions?”

This investigation uses a computer spreadsheet. You might want to check out the “Spreadsheet Tune Up” that begins on page 440 of the textbook. It works through some sample spreadsheets to help you remember your spreadsheet skills.

1. Turn to pages 168 and 169 in your textbook. Answer questions 1 to 3 of “Investigation 1: What happens to area when you change the dimensions?”

   Spreadsheet template Mod_3_1.xlt, found in the Spreadsheets folder on the multimedia CD, is similar to the spreadsheet from Investigation 1.

Check your answers on pages 76 to 78 in the Appendix.



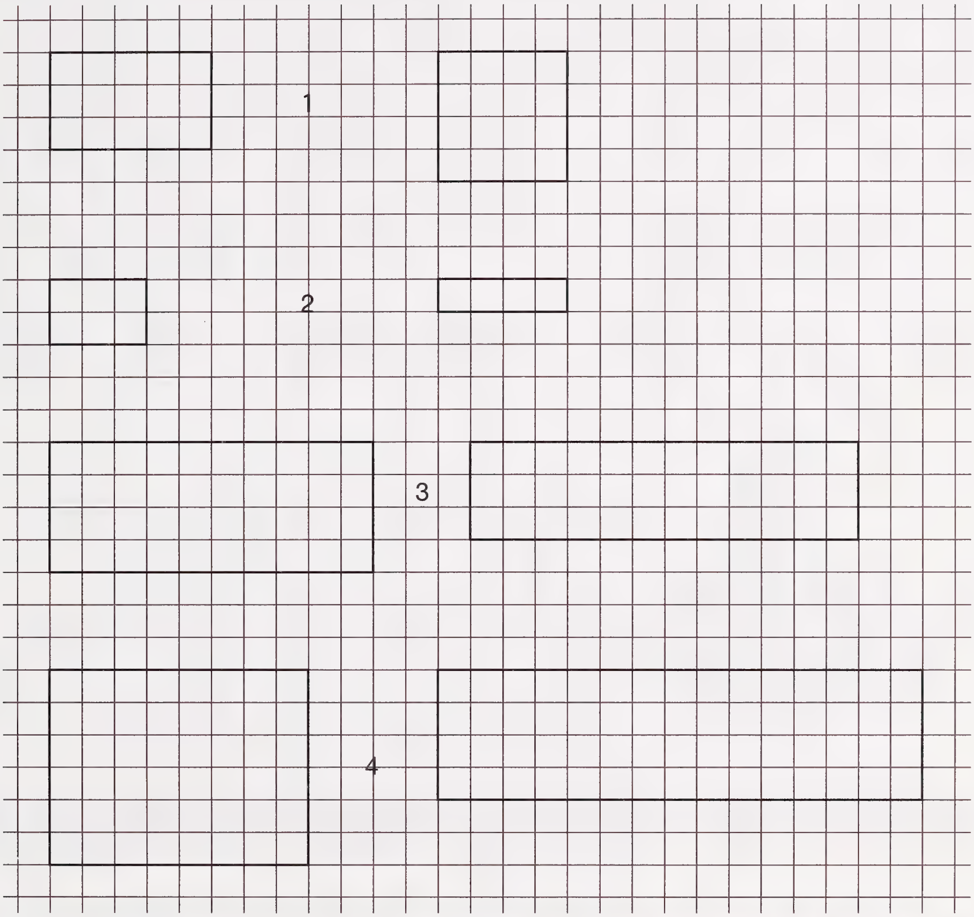
The next investigation also uses a computer spreadsheet. This one is very similar to the one you built for Investigation 1.

2. Turn to pages 169 and 170 in the textbook. Answer questions 1 to 3 of “Investigation 2: Changing Perimeter.”

   Spreadsheet template Mod_3_2.xlt, found in the Spreadsheets folder on the multimedia CD, is similar to the spreadsheet from Investigation 2.

Check your answers on pages 79 to 82 in the Appendix.

3. Answer questions 1 to 4 of “Investigation 3: Perimeter and Area Patterns” on page 170 in the textbook. Use the following shapes. There are two shapes for each question.

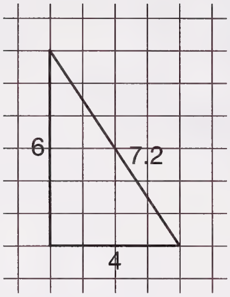


Check your answers on pages 82 to 84 in the Appendix.

Example

What happens to the perimeter and area of the following triangle if the lengths of its sides are doubled? What happens if they are halved?

The area of the original triangle is

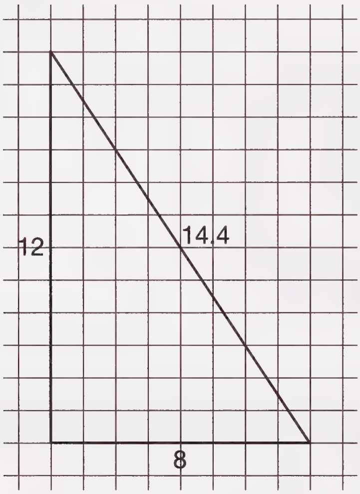


$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 4 \times 6$$
$$= 12$$

The perimeter of the original triangle is

$$P = 4 + 6 + 7.2$$
$$= 17.2$$

Doubling the lengths of the sides gives the following triangle.



The area of this triangle is

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 8 \times 12$$
$$= 48$$

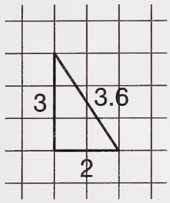
This area is 4 times the original area.

The perimeter of this triangle is

$$P = 8 + 12 + 14.4$$
$$= 34.4$$

This perimeter is 2 times the original perimeter.

Halving the lengths of the sides of the original triangle gives the following triangle.



The area of this triangle is

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 2 \times 3 \\ &= 3 \end{aligned}$$

This area is one quarter of the original area.

The perimeter of this triangle is

$$\begin{aligned} P &= 2 + 3 + 3.6 \\ &= 8.6 \end{aligned}$$

This perimeter is one half of the original perimeter.

You've investigated how perimeter and area relate to each other. Now it's time to put that work to use in solving some practical problems.

4. Turn to page 171 in your textbook and answer questions 1 to 3 of "Put into Practice."

Check your answers on pages 85 to 87 in the Appendix.

In this lesson you studied the relationships between the area and perimeter of various shapes.

Turn to the Section 2 Assignment in Assignment Booklet 3A. Answer question 3.

CONCLUSION

In Section 2 you used the area of objects to solve problems. In Section 2 you used the areas of a few simple shapes to find the areas of much more complicated shapes. You saw that knowing about the area of two-dimensional shapes let you find the surface area of three-dimensional shapes. You also saw that area and perimeter are not connected by an obvious relationship.

While the concepts of perimeter and area are something only people can really understand, other animals encounter them in their lives too. Racehorses have a practical knowledge of perimeter (how far around the track they have to run). They also know a lot about area (how large their stall is in the barn, for example). These horses spend their lives enclosed in paddocks and stalls and pens. They have enough room (area) for living and enough room (perimeter) for running. They're just like other farm animals, enclosed by fences built to protect them and keep them from wandering away.

SECTION 3

Measurement and Volume

A hot-air balloon floating across the sky is an impressive sight. It hardly seems possible that such a large object could float in the air. In order for a hot-air balloon to be able to carry a load of people, it has to be large. In fact, just being able to lift its own weight requires a fairly substantial balloon.

A hot-air balloon floats because the warm air inside the balloon is less dense than the cooler air outside. How much a balloon can lift is mostly determined by how large it is (the volume of air inside it) and how large the temperature difference is between the inside and outside air. A hot-air balloon the size of a garbage bag might be able to lift only 50 or 60 grams.

In this section you will investigate how to find the **volume** of objects. You will also look at different ways of measuring the same quantities.

LESSON 1

Enlargements and Reductions

In this lesson you will learn about **enlargements** and **reductions**.

Have you ever seen a set of Russian nesting dolls (*matryoshka*)? Most of them have either eight or five wooden dolls that fit one inside the other. The little ones seem to be shrunken copies of the larger ones.

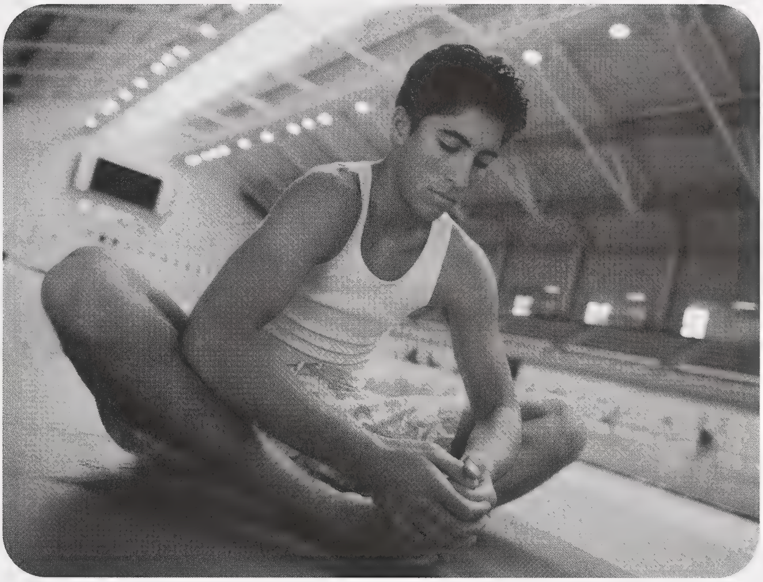
Turn to page 172 in the textbook. Read the "get thinking" box at the top of the page. Then continue by working through "Investigation: How do they compare?"

1. Answer questions 1 to 6 of "Investigation: How do they compare?" on pages 172 to 174 in the textbook.

Check your answers on pages 87 to 90 in the Appendix.

Example

Ethan is preparing to travel from Hanna to Oyen for a gymnastics competition. His map has a **scale** of 1:1 500 000. He measures the distance between Hanna and Oyen on the map as 6.7 cm. About how far apart are the actual towns?



This can be solved using a **proportion**. Let *x* be the actual distance between the two towns.

$$\frac{1}{1\,500\,000} = \frac{6.7\text{ cm}}{x}$$

$$\frac{x}{6.7\text{ cm}} = \frac{1\,500\,000}{1} \quad \text{Flip the equation.}$$

$$\cancel{6.7\text{ cm}} \times \frac{x}{\cancel{6.7\text{ cm}}} = 6.7\text{ cm} \times \frac{1\,500\,000}{1}$$

$$x = 10\,050\,000\text{ cm}$$

The calculated distance is 10 050 000 cm.

Normally distances between towns and cities are given in kilometres, so a conversion has to be done.

$$10\,050\,000\text{ cm} = \overset{100\,500}{\cancel{10\,050\,000\text{ cm}}} \times \frac{1\text{ m}}{\underset{1}{\cancel{100\text{ cm}}}}$$ First, change centimetres to metres.

$$= 100\,500\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}}$$ Next, change metres to kilometres.

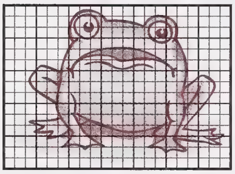
$$= \frac{100\,500\text{ km}}{1000}$$ Finally, simplify.

$$= 100.5\text{ km}$$

The distance between Oyen and Hanna is about 101 km.

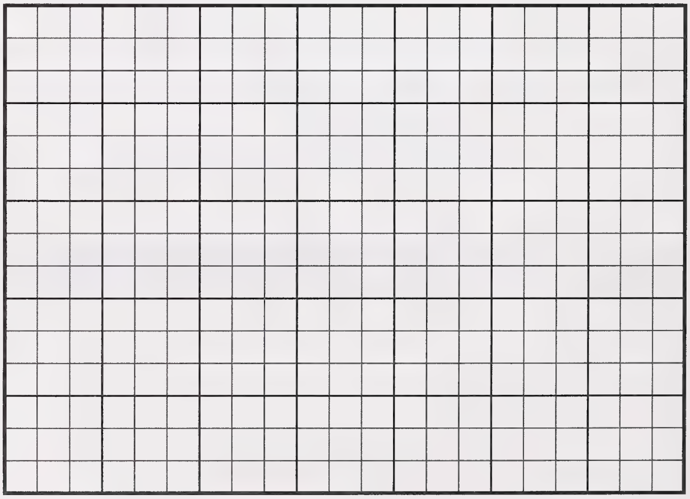
Example

Enlarge this frog by a factor of 3.

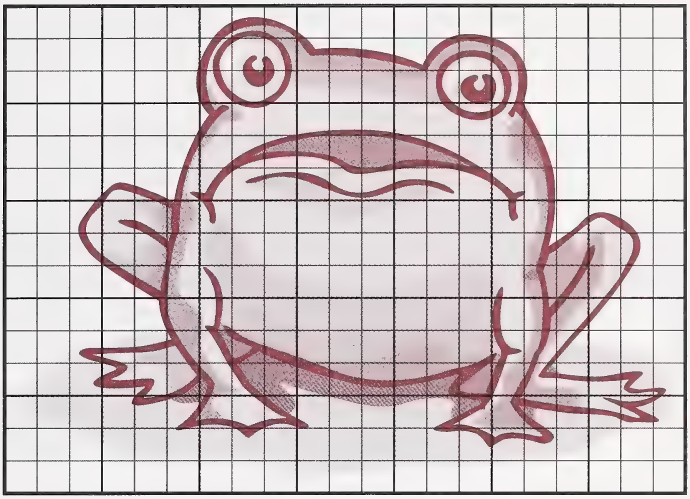


The first step is to draw an evenly spaced grid over the picture of the frog.

The next step is to make a grid that is three times larger.



The final step is to carefully copy each square from the small grid to the large grid.



The result is the enlarged frog.

Now you get to work through some scale problems yourself.

2. Turn to page 174 in your textbook and answer questions 1 to 5 of "Put into Practice."

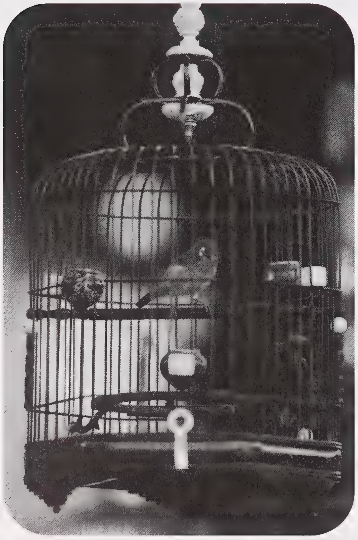
Check your answers on pages 90 and 91 in the Appendix.

In this lesson you studied enlargements and reductions, the use of scale drawings, and maps.

Turn to the Section 3 Assignment in Assignment Booklet 3B. Answer question 1.

LESSON 2

Solving Volume Problems



In this lesson you will learn about solving problems involving the volume of objects.

In the wild, birds live a free life, with few restrictions on their movements. They have an almost unlimited amount of space to move about in. The pet bird in the photo has a much more restricted lifestyle. It has a very limited amount of space to use.

Turn to page 176 in the textbook. Read the "get thinking" box at the top of the page. Then continue by working through "Investigation 1: Can you develop a formula for determining volume?"

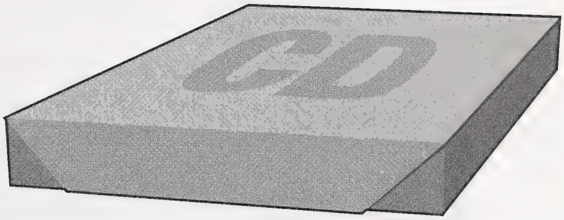
1. Answer questions 1 to 3 of "Investigation 1: Can you develop a formula for determining volume?" on pages 176 and 177 in the textbook.

Check your answers on pages 91 and 92 in the Appendix.

Example

A box used to ship CDs to mail-order customers measures 18 cm by 14 cm by 3 cm. What is the volume of the box?

The area of the base of the box is 18 cm × 14 cm = 252 cm^2. The volume of the box can be found as follows:



$$\begin{aligned} V &= A \times h \\ &= 252\text{ cm}^2 \times 3\text{ cm} \\ &= 756\text{ cm}^3 \end{aligned}$$

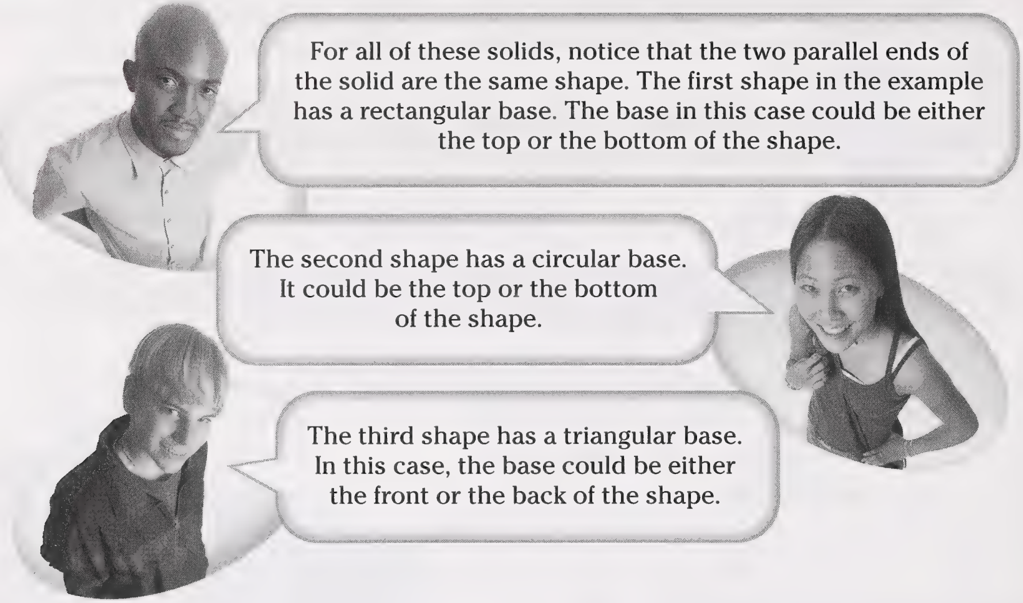
The volume of the box is 756 cm^3.

Turn to pages 177 to 179 in the textbook. Read through "Investigation 2: Volumes of Other **Prisms**." Make sure that you read the reminders on pages 178 and 179.

2. Answer questions 1 to 4 of "Investigation 2: Volumes of Other Prisms" on pages 177 to 179 in the textbook.

Check your answers on pages 92 and 93 in the Appendix.

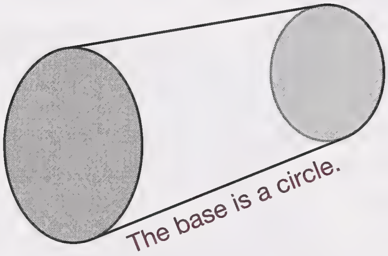
Turn to page 180 in the textbook and study the example carefully. It shows a simple procedure that can be used to find the volume of solids. It shows how to handle solids with bases that are rectangles, circles, and triangles.



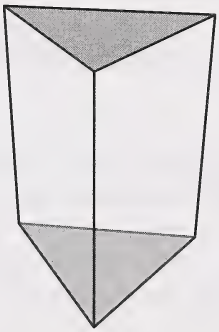
Example

What shape is the base of each of these solids?

a.

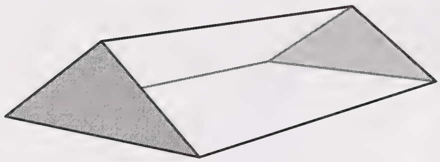


b.



The base is a triangle.

c.



The base is a triangle.

Notice that the base does not have to be the side that the shape is sitting on.

3. Turn to pages 181 and 182 in the textbook and answer questions 1 to 6 of "Put into Practice."

Check your answers on pages 93 to 96 in the Appendix.

In this lesson you saw how to find the volume of solids.

Turn to the Section 3 Assignment in Assignment Booklet 3B. Answer question 2.

LESSON 3

Other Types of Measurement

In this lesson you will learn about other types of measurements.

Which of the containers in the photo has the most fluid in it? Without being able to see the markings on the containers, it is hard to tell. The different shapes and sizes make it hard to compare the amount of fluid each contains. Yet, if each of the containers was marked in millilitres, you would be able to tell very quickly which holds the most.

Turn to page 183 in the textbook. Read the "get thinking" box at the top of the page. Then continue by working through "Investigation: SI Units."

1. Answer questions i. to vii. of "Investigation: SI Units" on page 183 in the textbook.

Check your answers on page 97 in the Appendix.

Turn to page 184 in the textbook and study the example closely. Read the reminder for the example as well.

Example

Cat food is on special at \$7.89 for a 9-kg bag. What is the unit price (dollars per kilogram) of this cat food?

Let x be the cost of 1 kg of cat food. Solving the following proportion will give a value for x.

$$\frac{x}{\$7.89} = \frac{1}{9}$$

$$\cancel{\$7.89} \times \frac{x}{\cancel{\$7.89}} = \$7.89 \times \frac{1}{9}$$

$$x \doteq \$0.876\ 666\ 666\ 7$$

The unit price of this cat food is \$0.88/kg.

Example

The daily recommended dose of a vitamin is 17 mg. A cereal package says one serving gives 40% of the daily recommended dose. How many milligrams of this vitamin are in one serving of cereal?

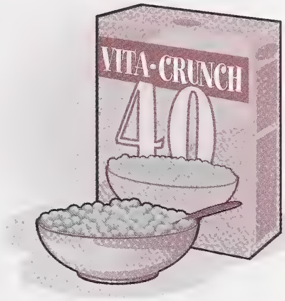
Let x be the number of milligrams of vitamin in a serving of cereal. Solving the following proportion will give a value for x.

$$\frac{x}{17 \text{ mg}} = 40\%$$

$$\frac{x}{17 \text{ mg}} = \frac{40}{100}$$

$$\cancel{17 \text{ mg}} \times \frac{x}{\cancel{17 \text{ mg}}} = 17 \text{ mg} \times \frac{40}{100}$$

$$x = 6.8 \text{ mg}$$



There are 6.8 mg of the vitamin in one serving of cereal.

2. Turn to pages 184 to 187 in the textbook and answer questions 1, 2, 3, 5, 7, and 9 of "Put into Practice."

Check your answers on pages 97 to 99 in the Appendix.

In this lesson you studied different ways of measuring the same quantities.

Turn to the Section 3 Assignment in Assignment Booklet 3B. Answer question 3.

CONCLUSION

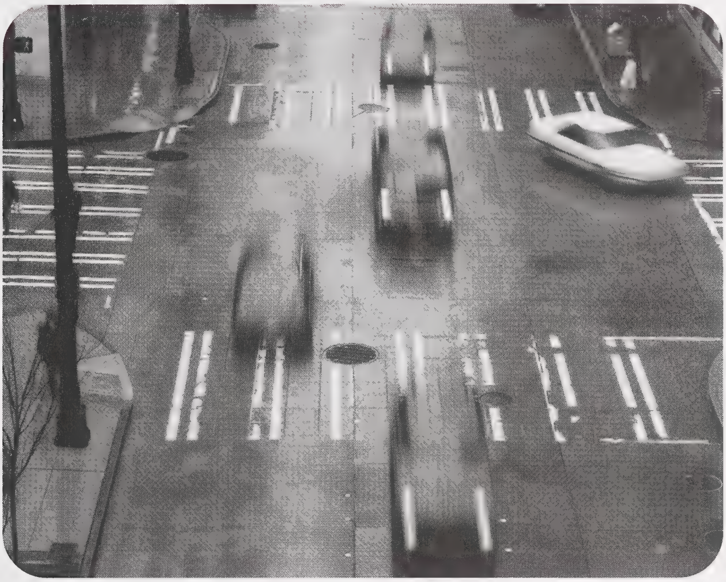
In this section you learned about enlargements and reductions. You solved problems using scale factors and percentages. You studied volumes of solids. You saw how changing all dimensions by a given factor affected the area and volume of the shape.

It takes a large volume of warm air to make a hot-air balloon lift off from Earth. Once they are airborne, they just follow the winds. It may not be the fastest way to travel from city to city, but it is an amazing experience.

If you want to learn more about how hot-air balloons work and have access to the Internet, try the following website:

http://travel.howstuffworks.com/hot-air-balloon.htm

MODULE SUMMARY



Throughout this module you used measurement and geometry to solve problems. You saw that the perimeter of simple shapes was easy to find. You learned how to find the perimeter of irregular shapes, and how to find their areas. These skills were applied to solving real-world problems. You also used nets for solids to find their surface area. You used them to solve problems, like how much paint to buy to paint a room. You studied how to find the volume of solids. You also saw that there can be many different ways to state the same measurement.

The engine in Shay and Jack's truck might be called a 350 CID (cubic inch displacement) engine if it predated metric measurements of displacement. In any case, they were probably quite happy to get it working well and to get out on the road.

Turn to

Assignment Booklet 3B and complete the Final Module Assignment.

REVIEW

This Review will help you apply what you learned in Module 3 and prepare for the Final Test. Read the skills checklist for this module. Use this list to guide your study and to help you decide how much of the Review you should complete.

Skills Checklist

- ❏ Find the perimeter of shapes.
- ❏ Find the circumference of circles.
- ❏ Find the area of rectangles, triangles, and parallelograms.
- ❏ Find the area of circles.
- ❏ Solve problems about the area of two-dimensional shapes.
- ❏ Solve problems about the surface area of three-dimensional shapes.
- ❏ Solve problems about the volume of objects.
- ❏ Relate area and perimeter of shapes.
- ❏ Use scale factors to
  - find distances on maps
  - analyze an enlargement of a shape
  - analyze a reduction of a shape
- ❏ Enlarge or reduce a drawing using squares of different sizes.
- ❏ Use different ways to measure objects.

Review Questions

Turn to pages 188 to 191 in your textbook. Answer questions 1 to 8 of "Review of Unit Three." In question 5 you can choose a different scale (1 cm:2 dm, for example) if you cannot fit your net on a standard sheet of paper.

Check your answers on pages 99 to 107 in the Appendix.

MATHEMATICS 24

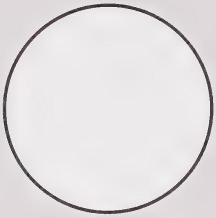
Appendix

GLOSSARY
ANSWER KEY
IMAGE CREDITS
LEARNING AIDS

Glossary

area: the size of a surface measured in square units

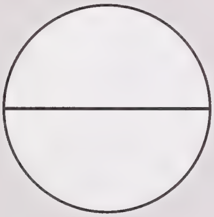
circle: a curve where all points are the same distance from the centre



circumference: the perimeter of a circle

cylinder: a three-dimensional solid with two parallel congruent circles as ends

diameter: a line joining two points on a circle and passing through the centre of the circle

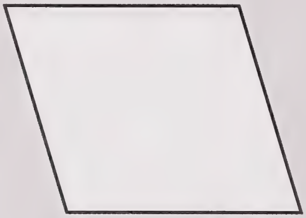


enlargement: a copy that is larger than the original

net: a pattern that can be folded to make a three-dimensional shape

A net is useful in finding the surface area of a solid.

parallelogram: a four-sided shape with parallel opposite sides



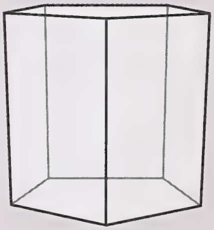
perimeter: the length of the boundary of a closed figure

pi: the ratio of the circumference of a circle to its radius

It has a value of about 3.141 592 654.

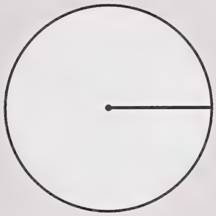
prism: a three-dimensional figure with two parallel congruent polygons as bases

The sides are rectangles.

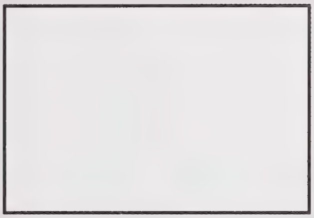


proportion: an equation showing that two ratios are equal

radius: a line segment that joins the centre of a circle with a point on its circumference; also, the length of such a line segment

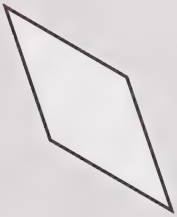


rectangle: a four-sided figure with opposite sides equal and parallel



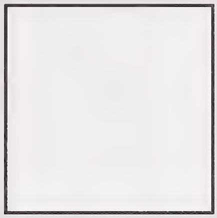
reduction: a copy that is smaller than the original

rhombus: a parallelogram with four equal sides



scale: the size of a plan, map, drawing, or model compared with what it represents

square: a rhombus with all angles equal to 90°



surface area: the sum of the areas of the faces of a three-dimensional object

triangle: a three-sided figure

volume: the amount of space occupied by an object

Answer Key

Section 1: Perimeter and Area

Lesson 1: Using Perimeter to Solve Problems

1. Textbook, page 143, "Put into Practice," questions 1.b., 1.c., 2.b., 2.c., and 2.e.

1. b. This shape is a square. All sides are equal. There are four sides. Each side is 6 inches long. To find the perimeter, you can use either addition or multiplication. (Remember that multiplication is really just repeated addition.)

$$6 \text{ in} + 6 \text{ in} + 6 \text{ in} + 6 \text{ in} = 24 \text{ in}$$

$$6 \text{ in} \times 4 = 24 \text{ in}$$

Both methods show that the perimeter is 24 inches.

c. This shape is a rectangle. There are two sides that are 15 km long. The other two sides are 4.3 km long. To find the perimeter, you can use either addition or addition and multiplication.

$$15 \text{ km} + 15 \text{ km} + 4.3 \text{ km} + 4.3 \text{ km} = 38.6 \text{ km}$$

$$\begin{aligned} 2 \times (15 \text{ km} + 4.3 \text{ km}) &= 2 \times 19.3 \text{ km} \\ &= 38.6 \text{ km} \end{aligned}$$

Both methods show that the perimeter is 38.6 km.

2. b. This shape is a trapezoid. There are two equal sides. (You know this because they are marked with the same symbol.) These sides are 15 m long. The two parallel sides are 26.75 m and 16.5 m long. To find the perimeter, you can use either addition or addition and multiplication.

$$15 \text{ m} + 15 \text{ m} + 26.75 \text{ m} + 16.5 \text{ m} = 73.25 \text{ m}$$

$$\begin{aligned} (2 \times 15 \text{ m}) + 26.75 \text{ m} + 16.5 \text{ m} &= 30 \text{ m} + 26.75 \text{ m} + 16.5 \text{ m} \\ &= 73.25 \text{ m} \end{aligned}$$

Both methods show that the perimeter is 73.25 m.

c. This shape is a regular octagon. There are eight equal sides. (All of the sides are marked with the same symbol.) The easiest way to find the perimeter is to use multiplication. You could use addition as well.

$8 \times 4.8 \text{ cm} = 38.4 \text{ cm}$

$4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} + 4.8 \text{ cm} = 38.4 \text{ cm}$

The perimeter is 38.4 cm.

e. This shape is a triangle. You have to add the lengths of the sides to find the perimeter.

$36 \text{ in} + 90 \text{ in} + 82.5 \text{ in} = 208.5 \text{ in}$

The perimeter is 208.5 inches.

2. Textbook, page 144, "Put into Practice," questions 3, 4.a., and 4.c.

3. You can estimate the circumference of a circle by multiplying the diameter by 3.

4. a. The diagram shows a circle with a diameter of 5 m. The circumference is the product of the diameter and π.

Estimate

$$\begin{aligned} C &= \pi d \\ &\doteq 3 \times 5 \text{ m} \\ &\doteq 15 \text{ m} \end{aligned}$$

An estimated value for the circumference is 15 m.

Calculate

$$\begin{aligned} C &= \pi d \\ &= \pi \times 5 \text{ m} \\ &\doteq 15.707\,963\,27 \text{ m} \end{aligned}$$

The circumference, rounded to one decimal place, is 15.7 m.

c. The diagram shows a circle with a radius of 9.32 ft. The circumference is the product of twice the radius and π.

Estimate

$$\begin{aligned} C &= 2\pi r \\ &\doteq 2 \times 3 \times 9 \text{ ft} \\ &\doteq 54 \text{ ft} \end{aligned}$$

An estimated value for the circumference is 54 ft.

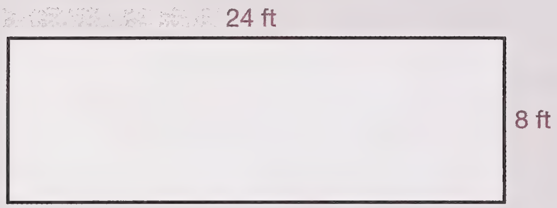
Calculate

$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 9.32 \text{ ft} \\ &\doteq 58.559\,287\,06 \text{ ft} \end{aligned}$$

The circumference, rounded to one decimal place, is 58.6 ft.

3. **Textbook, page 145, "Put into Practice," questions 5 and 6**

5. a.



b. The perimeter is the sum of the lengths of the sides.

$$\begin{aligned} P &= 24\text{ ft} + 24\text{ ft} + 8\text{ ft} + 8\text{ ft} \\ &= 64\text{ ft} \end{aligned}$$

The perimeter of the kennel is 64 ft.

c. George needs to buy 2.5 ft less fencing than the perimeter of the kennel.

$$\begin{aligned} \text{length} &= 64\text{ ft} - 2.5\text{ ft} \\ &= 61.5\text{ ft} \end{aligned}$$

George will have to buy 61.5 ft of chain-link fencing.

d. The total cost of the chain-link fencing is the product of its price per foot and the number of feet needed.

$$\begin{aligned} \text{cost} &= \$1.32 \times 61.5 \\ &= \$81.18 \end{aligned}$$

The cost of the chain-link fencing is $81.18.

e. First, George will need to find the cost of the posts.

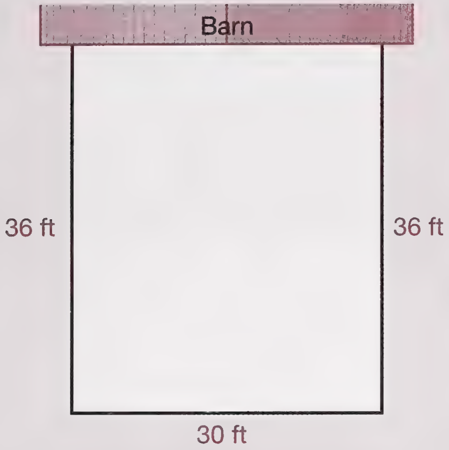
$$\begin{aligned} \text{posts} &= 16 \times \$2.54 \\ &= \$40.64 \end{aligned}$$

Then he will have to add the post cost, the fencing cost, and the gate cost.

$$\begin{aligned} \text{total} &= \$40.64 + \$81.18 + \$56.00 \\ &= \$177.82 \end{aligned}$$

George's cost, before tax, will be $177.82.

6. **a.** The new fence is made up of two sides of 36 ft and one side of 30 ft.



$$\begin{aligned} \text{length} &= 36\text{ ft} + 36\text{ ft} + 30\text{ ft} \\ &= 102\text{ ft} \end{aligned}$$

Suzette's new fencing will have a total length of 102 ft.

b. Suzette will have to divide the length of fencing by 6 to see how many 6-ft sections she will need.

$$\begin{aligned} \text{sections of fence} &= 102\text{ ft} \div 6\text{ ft} \\ &= 17 \end{aligned}$$

Each section takes three fence boards.

$$\begin{aligned} \text{number of boards} &= 3 \times 17 \\ &= 51 \end{aligned}$$

Suzette will need 51 boards to build her fence.

c. Suzette will have to multiply the number of boards by the cost per board to find her before-tax cost.

$$\begin{aligned} \text{cost} &= 51 \times \$2.31 \\ &= \$117.81 \end{aligned}$$

The fence boards will cost Suzette \$117.81.

Lesson 2: Areas of Rectangles, Triangles, and Parallelograms

1. Textbook, pages 147 and 148, "Investigation 1: How can you determine the areas of rectangular shapes?", questions 2 to 6

2.

Rectangle	Base (cm)	Height (cm)	Number of Squares in the Rectangle	Area of the Rectangle (cm^2)	Perimeter (cm)
A	5	3	15	15	16
B	6	4	24	24	20
C	2	13	26	26	30
D	6	11	66	66	34

3. Area is the number of squares needed to cover the surface of the shape.

4. a. The product of the base and the height gives the area.
b. $A = b \times h$

5. a. Area is how much surface, while perimeter is how far around.
b. The units for area are squares. Some examples are cm^2, m^2, in^2, ft^2, and km^2.

6. a. perimeter: The thread to be used would be related to the lengths of the sides of the tablecloth.

b. area: The seed has to cover the surface of the new lawn.

c. perimeter: The border would be related to the lengths of the sides of the flower bed.

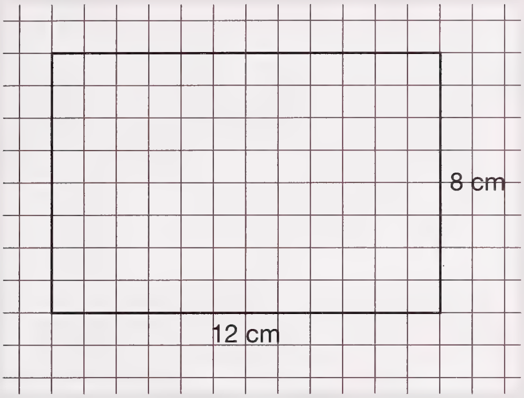
d. area: The surface of the fence is needed to know how much paint to purchase.

e. both: You need to know the perimeter in order to buy enough fencing material. You need the area to make sure the dog has enough space.

f. both and more: You would need the perimeter to know how much material to purchase in order to build the forms to hold the concrete. You need the area and the depth of concrete to know how much concrete to order.

2. **Textbook, pages 148 and 149, "Investigation 2: How can you determine the areas of triangles?", questions 1 to 3**

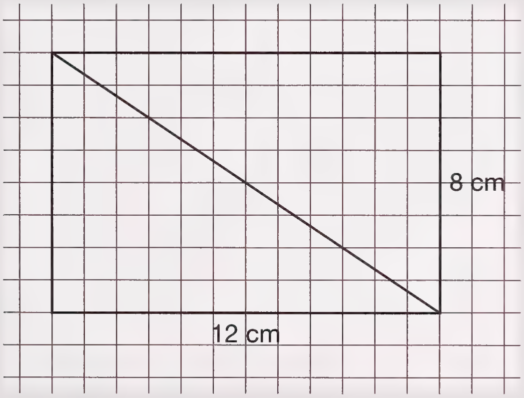
1. **a.** Your drawing should look like the one that follows.



b. The area is 96 cm^2.

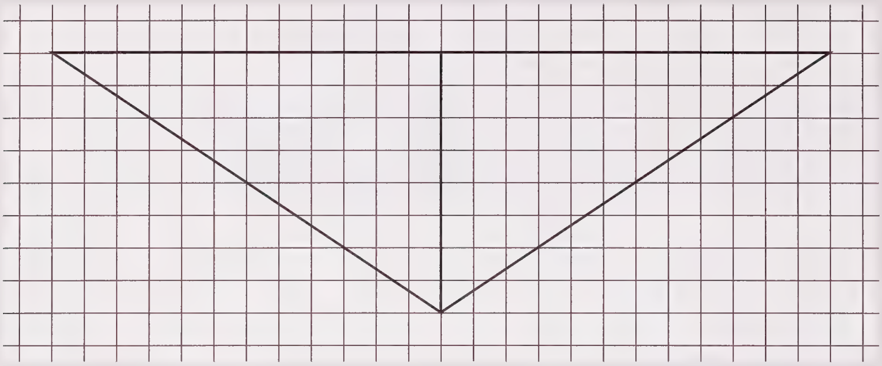
$12 \times 8 = 96$

2. **a.** Your drawing should look like the one that follows. Your diagonal may go the other way.

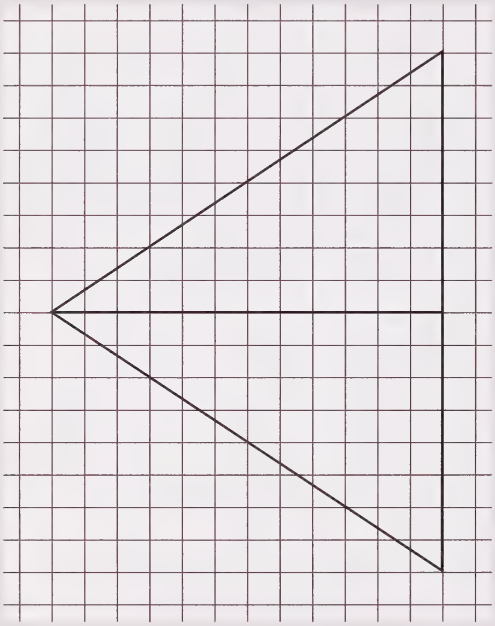


b. The resulting shapes are triangles.

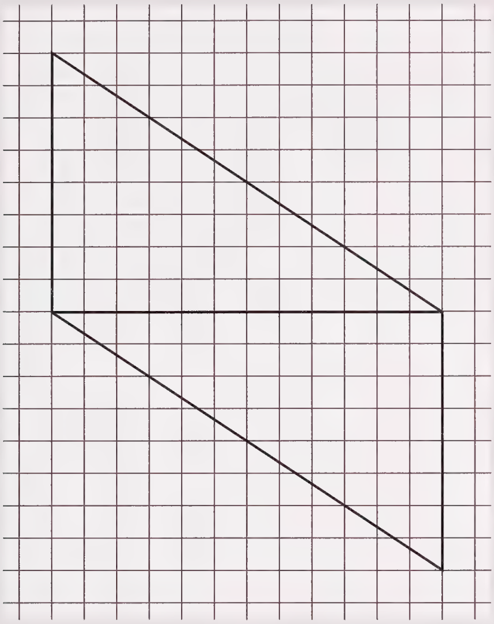
c. Yes, you can combine the two shapes to make a triangle. Each small triangle has an area of 48 cm^2. The large triangle has an area of 96 cm^2.



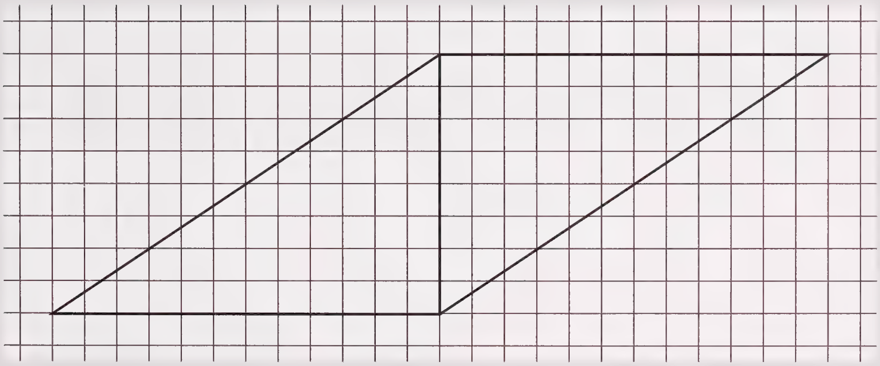
OR



d. Yes, you can combine the shapes to make a parallelogram. Each parallelogram has an area of 96 cm^2.

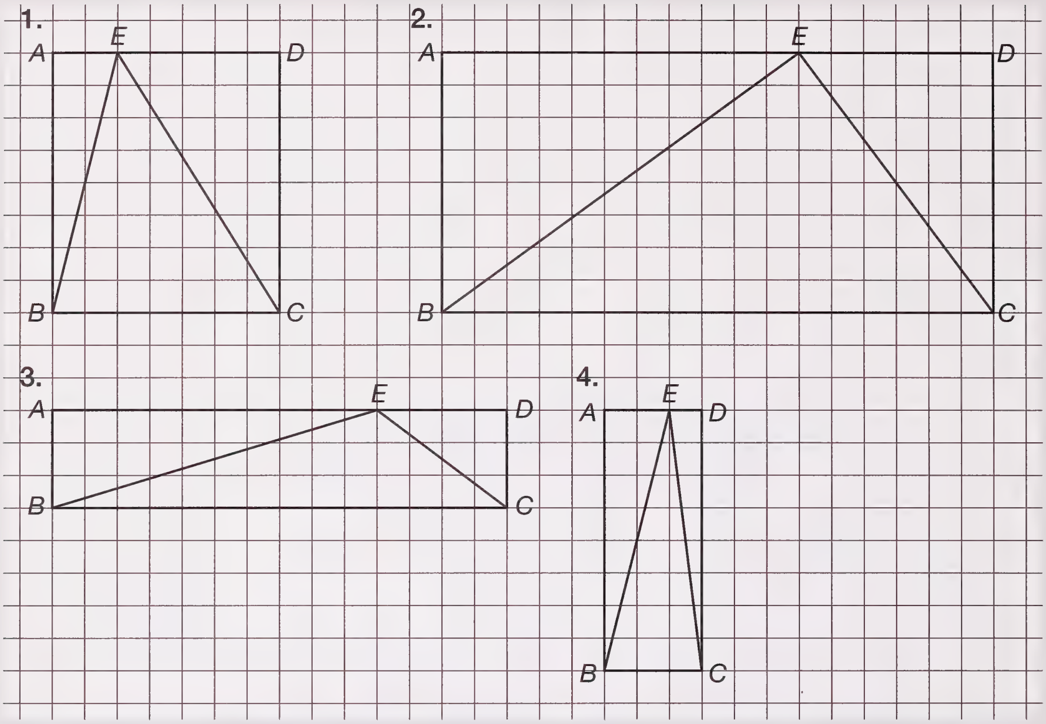


OR



e. Yes, it is possible to use the shapes to make a kite. The area will be 96 cm^2.

3. a. Answers will vary. A sample answer follows.



	Rectangle *ABCD*			Triangle *EBC*		
	Base *BC* (cm)	**Height *AB* (cm)**	**Area (cm^2)**	**Base *BC* (cm)**	**Height (cm)**	**Area (cm^2)**
1.	7	8	56	7	8	28
2.	17	8	136	17	8	68
3.	14	3	42	14	3	21
4.	3	8	24	3	8	12

b. Half of 56 is 28; half of 136 is 68; half of 42 is 21; and half of 24 is 12.

The area of each triangle is half of the area of the corresponding rectangle.

c. $A = \frac{1}{2} b \times h$

3. **Textbook, page 152, "Put into Practice," questions 1.b., 1.c., 2.c., and 2.e.**

1. b. The shape is a square with all sides measuring 6 inches. (All sides are marked with the same symbol.)

$$\begin{aligned} A &= bh \\ &= 6 \times 6 \\ &= 36 \end{aligned}$$

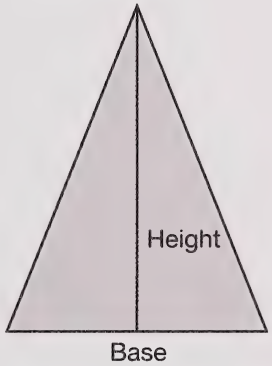
The area of the square is 36 in^2.

c. The rectangle has sides measuring 15 km and 4.3 km.

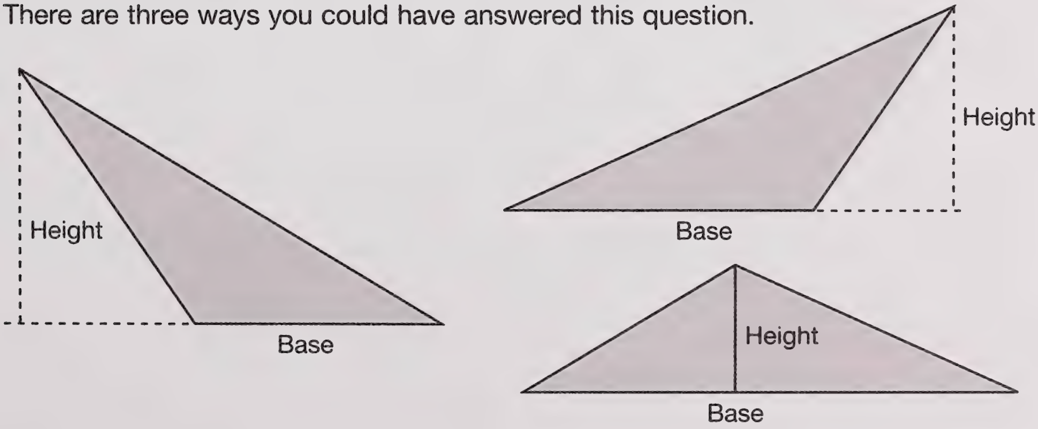
$$\begin{aligned} A &= bh \\ &= 15 \times 4.3 \\ &= 64.5 \end{aligned}$$

The area of the square is 64.5 km^2.

2. c. There are three ways you could have answered this question. This is the most obvious way.

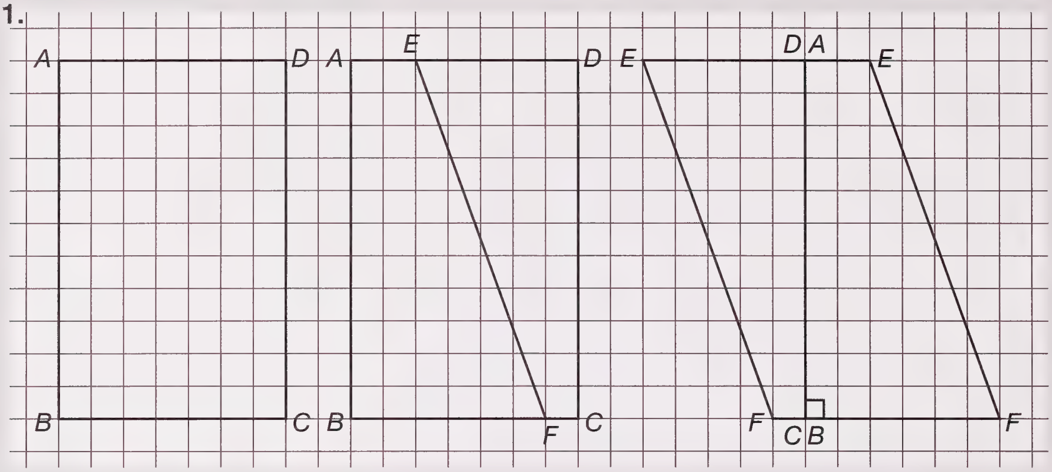


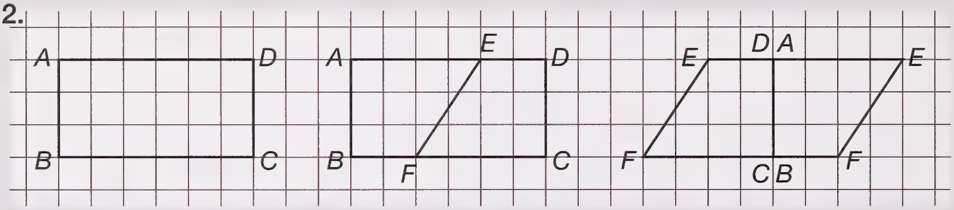
e. There are three ways you could have answered this question.



4. **Textbook, page 150, "Investigation 3: Determining Areas of More Parallelograms," question 1**

1. **a.** Answers will vary. A sample answer follows.





	Rectangle			Parallelogram		
	Base (cm)	Height (cm)	Area (cm^2)	Base (cm)	Height (cm)	Area (cm^2)
1.	7	11	77	7	11	77
2.	6	3	18	6	3	18

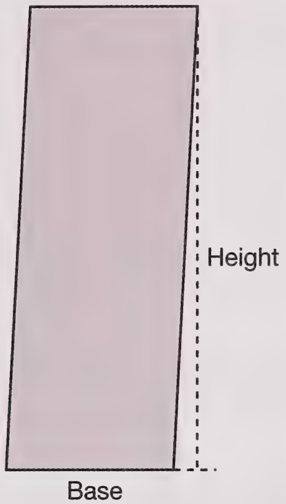
b. Seventy-seven is the same as 77; 18 is the same as 18.

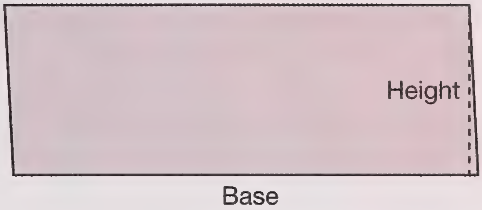
The areas of a rectangle and a parallelogram with the same base and height are the same.

c. $A = b \times h$

5. Textbook, page 153, "Put into Practice," questions 2.b., 3.a., 3.b., and 3.d.

2. b. There are two possible answers.





3. a. This shape is a parallelogram. The horizontal sides are 14 inches long. The sloped sides are 7 inches long. The height is 5 inches.

Estimate

$$\begin{aligned} A &= bh \\ &\doteq 10 \text{ in} \times 5 \text{ in} \\ &\doteq 50 \text{ in}^2 \end{aligned}$$

An estimated area for this parallelogram is 50 in^2.

Calculate

$$\begin{aligned} A &= bh \\ &= 14 \text{ in} \times 5 \text{ in} \\ &= 70 \text{ in}^2 \end{aligned}$$

The area of this parallelogram is 70 in^2.

b. This shape is a parallelogram. The horizontal sides are 3.2 m long. The sloped sides are 9.3 m long. The height is 8.1 m.

Estimate

$$\begin{aligned} A &= bh \\ &= 3 \text{ m} \times 8 \text{ m} \\ &= 24 \text{ m}^2 \end{aligned}$$

An estimated area for this parallelogram is 24 in^2.

Calculate

$$\begin{aligned} A &= bh \\ &= 3.2 \text{ m} \times 8.1 \text{ m} \\ &= 25.92 \text{ m}^2 \end{aligned}$$

The area of this parallelogram, rounded to one decimal place, is 25.9 in^2.

d. This shape is an isosceles triangle. The two equal sides are 140 m. The third side is 113 m. The height is 128 m.

Estimate

$$A = \frac{1}{2}bh$$
$$\doteq \frac{1}{2} \times 100 \text{ m} \times 100 \text{ m}$$
$$\doteq 5000 \text{ m}^2$$

An estimated area for this triangle is 5000 m^2.

Calculate

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 113 \text{ m} \times 128 \text{ m}$$
$$= 7232 \text{ m}^2$$

The area of this triangle is 7332 m^2.

Lesson 3: Areas of Circles

1. Textbook, page 154, "Investigation: How can you estimate the area of a circle?", questions 1 to 3

1. Answers will vary. A sample answer follows.

Circle	Radius (cm)	(Radius)^2 (cm^2)	Estimated Area by Counting Squares (cm^2)
A	1	1	3 It covers about $\frac{3}{4}$ of 4 squares.
B	2	4	12 It covers 4 complete squares and most of 8 more. The missing parts are made up for by parts of 4 other squares.
C	3	9	28 It covers 16 complete squares and most of 12 more. The missing parts are made up for by parts of 4 other squares.

2. The area seems to be about 3 times the radius squared.

3. To estimate the area of a circle, multiply the square of the radius by 3.

2. Textbook, page 156, "Put into Practice," questions 1.b., 1.c., and 2

1. b. A circle with a radius of 4.9 cm is shown in the diagram in the textbook.

Estimate

$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3 \times (5\text{ cm})^2 \\ &\doteq 75\text{ cm}^2 \end{aligned}$$

An estimated area for this circle is 75 cm^2.

Calculate

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (4.9\text{ cm})^2 \\ &\doteq 75.429\,639\,61\text{ cm}^2 \end{aligned}$$

The area of this circle, rounded to one decimal place, is 75.4 cm^2.

c. A circle with a diameter of 15 m is shown in the diagram in the textbook. This means the radius is 7.5 m (half of 15 m).

Estimate

$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3 \times (10\text{ m})^2 \\ &\doteq 300\text{ m}^2 \end{aligned}$$

An estimated area for this circle is 300 m^2.

Calculate

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (7.5\text{ m})^2 \\ &\doteq 176.714\,586\,8\text{ m}^2 \end{aligned}$$

The area of this circle, rounded to one decimal place, is 176.7 m^2.

2. The dog can play in an area shaped like a circle. This circle has a radius of 3.5 m. To find the area of the circular area, use the formula $A = \pi r^2$.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (3.5\text{ m})^2 \\ &\doteq 38.484\,510\,01\text{ m}^2 \end{aligned}$$

The dog can play in an area of 38.5 m^2.

Section 2: Area Problems

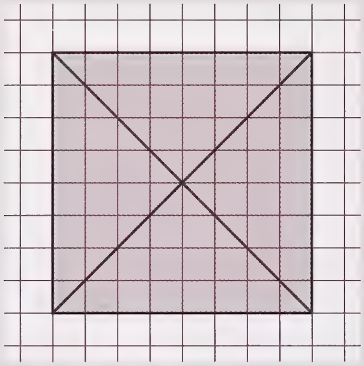
Lesson 1: Solving Area Problems with 2-D Shapes

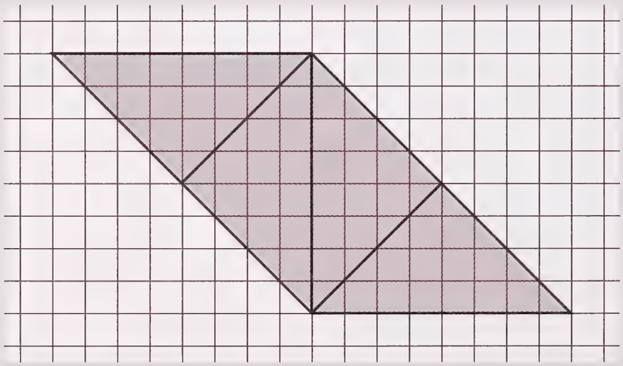
1. **Textbook, page 157, "Investigation 1: How can you determine the area of irregular or composite shapes?", questions 1 and 2**

   1. **a.** The area of the square is 100 cm^2.

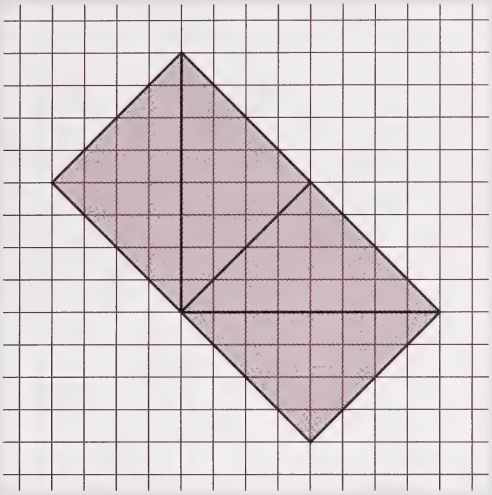
      b. Your results should look like the cut up square on the left side of page 157.

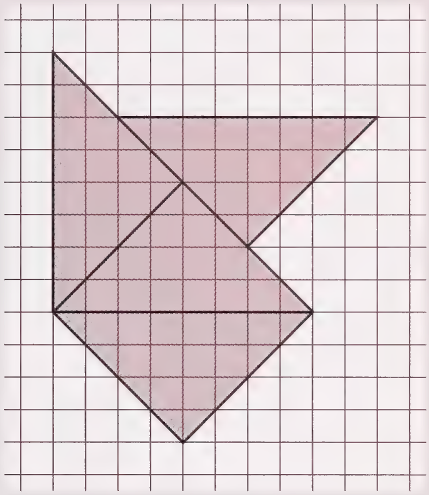
      c. The area of the parallelogram is also 100 cm^2. It may look like either of the following examples.

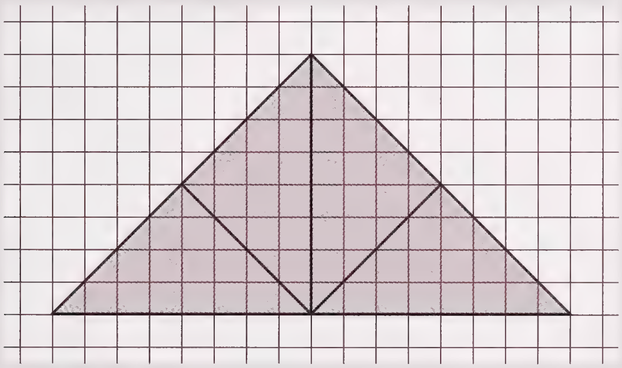




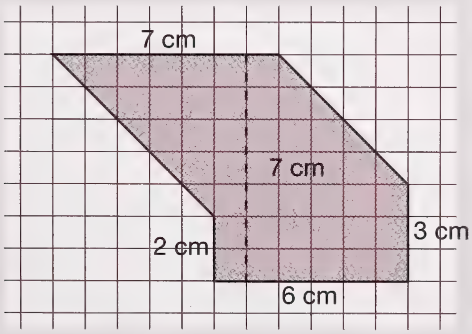
d. Answers will vary. The area of each shape is 100 cm^2.



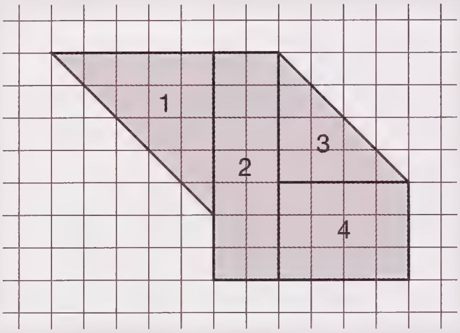


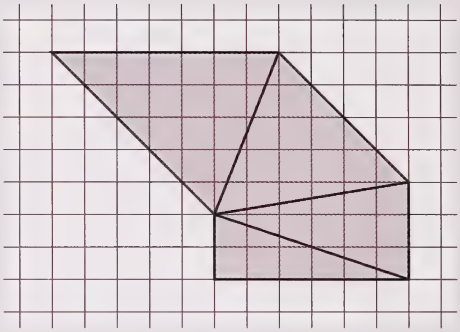


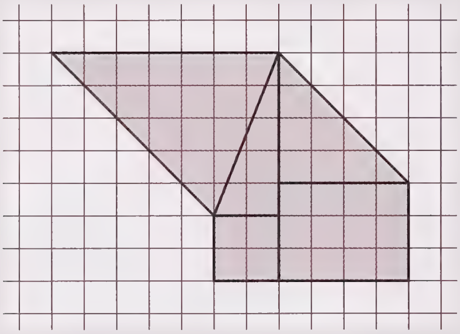
2. a. Answers will vary. A sample answer follows.



b. There are several ways to divide this shape into rectangles and/or triangles. Some ways follow.







c. The area of the parts is calculated only for the first sample above.

1

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 5\text{ cm} \times 5\text{ cm}$$
$$= 12.5\text{ cm}^2$$

2

$$A = b \times h$$
$$= 2\text{ cm} \times 7\text{ cm}$$
$$= 14\text{ cm}^2$$

3

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 4\text{ cm} \times 4\text{ cm}$$
$$= 8\text{ cm}^2$$

4

$$A = b \times h$$
$$= 4\text{ cm} \times 3\text{ cm}$$
$$= 12\text{ cm}^2$$

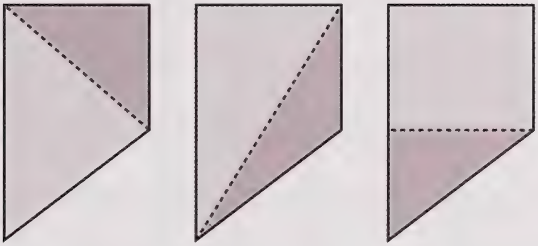
d. The total area of the shape is 46.5 cm^2.

$$12.5\text{ cm}^2 + 14\text{ cm}^2 + 8\text{ cm}^2 + 12\text{ cm}^2 = 46.5\text{ cm}^2$$

2. **Textbook, pages 159 to 163, "Put into Practice," questions 1.a., 1.b., 2.a., 2.c., 2.e., 2.f., 3.a., 3.b., 3.c., 3.d., 3.f., 3.h., 4, and 5**

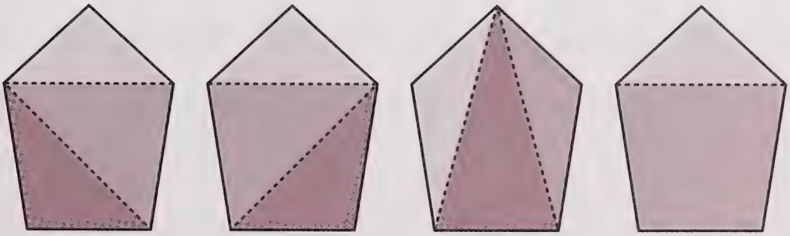
   1. **a.** and **b.** **i.** You could either make it the difference of two rectangles or the sum of two rectangles. Sample answers are shown.

ii. You could make this either the sum of two triangles or a triangle and a rectangle. Sample answers are shown.



iii. You could make this the sum of a rectangle and a semicircle. A sample answer is shown.

iv. You could make this either the sum of three triangles or a triangle and a trapezoid. Sample answers are shown.



2. a. This is a difference of a rectangle and a triangle.

   c. This is a difference of two rectangles.

   e. This is one quarter of a circle.

   f. This is both a sum and a difference. The house shape is the sum of a triangle and a rectangle. The door is a rectangle that has to be subtracted.

3. a. The area is the sum of a square measuring 12 m on each side and a semicircle with a diameter of 12 m. Since the base and height of a square are equal, the formula $A = s^2$ is often used instead of $A = bh$ when dealing with squares.

Square

$$A = s^2$$
$$= (12\text{ m})^2$$
$$= 144\text{ m}^2$$

Circle

$$A = \pi r^2$$
$$= \pi\left(\frac{12}{2}\text{ m}\right)^2$$
$$\doteq 113.097\ 335\ 5\text{ m}^2$$

Semicircle

$$A = \frac{1}{2} \times \text{area of circle}$$
$$\doteq \frac{1}{2} \times 113.097\ 335\ 5\text{ m}^2$$
$$\doteq 56.548\ 667\ 76\text{ m}^2$$

The shaded area, rounded to one decimal place, is $144\text{ m}^2 + 56.5\text{ m}^2 = 200.5\text{ m}^2$.

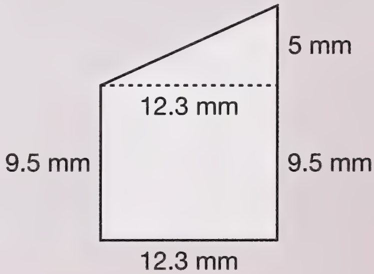
   b. The area is the sum of a triangle and a rectangle.

Rectangle

$$A = bh$$
$$= 12.3\text{ mm} \times 9.5\text{ mm}$$
$$= 116.85\text{ mm}^2$$

Triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 12.3\text{ mm} \times 5\text{ mm}$$
$$= 30.75\text{ mm}^2$$



The shaded area is $116.85\text{ mm}^2 + 30.75\text{ mm}^2 = 147.6\text{ mm}^2$.

c. This is the difference of a square and a semicircle.

Square

$$\begin{aligned} A &= s^2 \\ &= (10 \text{ in})^2 \\ &= 100 \text{ in}^2 \end{aligned}$$

Semicircle

$$\begin{aligned} A &= \frac{1}{2} \times \text{ area of circle} \\ &= \frac{1}{2}\pi r^2 \\ &= \frac{1}{2}\pi\left(\frac{10}{2} \text{ in}\right)^2 \\ &\doteq 39.269\,908\,17 \text{ in}^2 \end{aligned}$$

The shaded area, rounded to one decimal place, is $100 \text{ in}^2 - 39.3 \text{ in}^2 = 60.7 \text{ in}^2$.

d. This is the difference of two rectangles.

Rectangle 1

$$\begin{aligned} A &= bh \\ &= 23 \text{ cm} \times 18 \text{ cm} \\ &= 414 \text{ cm}^2 \end{aligned}$$

Rectangle 2

$$\begin{aligned} A &= bh \\ &= 13 \text{ cm} \times 8 \text{ cm} \\ &= 104 \text{ cm}^2 \end{aligned}$$

The shaded area is $414 \text{ cm}^2 - 104 \text{ cm}^2 = 310 \text{ cm}^2$.

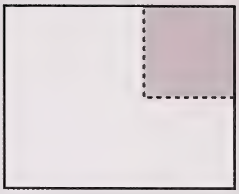
f. This is three quarters of a circle.

$$\begin{aligned} A &= \frac{3}{4} \times \text{area of circle} \\ &= \frac{3}{4}\pi r^2 \\ &= \frac{3}{4}\pi (8.6 \text{ m})^2 \\ &\doteq 174.264\,144\,5 \text{ m}^2 \end{aligned}$$

The shaded area, rounded to one decimal place, is 174.3 m^2.

h. One way is to find the difference of two rectangles.

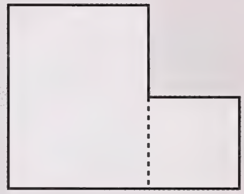
Rectangle 1	Rectangle 2
$A = bh$	$A = bh$
$= 280 \text{ cm} \times (215 \text{ cm} + 140 \text{ cm})$	$= 140 \text{ cm} \times 140 \text{ cm}$
$= 99\,400 \text{ cm}^2$	$= 19\,600 \text{ cm}^2$



The shaded area is $99\,400 \text{ cm}^2 - 19\,600 \text{ cm}^2 = 79\,800 \text{ cm}^2$.

Another way is to find the sum of two rectangles.

Rectangle 1	Rectangle 2
$A = bh$	$A = bh$
$= 280 \text{ cm} \times 215 \text{ cm}$	$= 140 \text{ cm} \times 140 \text{ cm}$
$= 60\,200 \text{ cm}^2$	$= 19\,600 \text{ cm}^2$



The shaded area is $60\,200 \text{ cm}^2 + 19\,600 \text{ cm}^2 = 79\,800 \text{ cm}^2$.

4. a. The ceiling is a rectangle.

$$\begin{aligned} A &= bh \\ &= 5.5 \text{ m} \times 4 \text{ m} \\ &= 22 \text{ m}^2 \end{aligned}$$

The area of the ceiling is 22 m^2.

b. The wall area to be painted is a large rectangle minus three smaller rectangles.

Walls	Door	Window
$A = bh$	$A = bh$	$A = bh$
$= 2.5 \text{ m} \times (5.5 \text{ m} + 4 \text{ m} + 5.5 \text{ m} + 4 \text{ m})$	$= 1 \text{ m} \times 2 \text{ m}$	$= 0.5 \text{ m} \times 1.25 \text{ m}$
$= 47.5 \text{ m}^2$	$= 2 \text{ m}^2$	$= 0.625 \text{ m}^2$

The area to be painted is the area of the walls minus the area of two doors and a window, or $47.5 \text{ m}^2 - 2 \text{ m}^2 - 2 \text{ m}^2 - 0.625 \text{ m}^2 = 42.875 \text{ m}^2$.

c. The total area to be painted is $22 \text{ m}^2 + 42.875 \text{ m}^2 = 64.875 \text{ m}^2$.

d. Let x be the number of litres of paint needed.

$$\frac{x}{64.875\text{ m}^2} = \frac{1\text{ L}}{9.5\text{ m}^2}$$

$$\frac{\cancel{64.875\text{ m}^2} \times x}{\cancel{64.875\text{ m}^2}} = \frac{64.875\,\cancel{\text{m}^2} \times 1\text{ L}}{9.5\,\cancel{\text{m}^2}}$$

$$x = \frac{64.875\text{ L}}{9.5}$$

$$x = 6.828\ 947\ 368\text{ L}$$

Jennifer and Wade will have to buy 7 L of paint.

e. They would need one 4-L can and three 1-L cans of paint. It would be slightly cheaper to buy two 4-L cans.

$$\begin{aligned}\text{cost} &= \$29.95 + (3 \times \$9.99)\\ &= \$59.92\end{aligned}$$

The paint would cost $59.92.

$$\begin{aligned}\text{cost} &= 2 \times \$29.95\\ &= \$59.90\end{aligned}$$

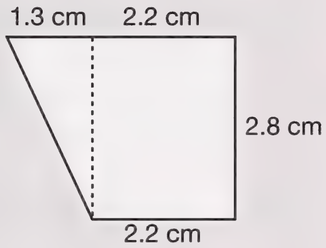
The paint would cost $59.90.

Which is the best solution (2¢ more cost or 1 L of wasted paint)?

5. The following diagram shows the measurements of the shape. It is divided into a rectangle and a triangle to make the area calculations.

$$\begin{aligned}A &= \frac{1}{2}bh\\ &= \frac{1}{2} \times 13\text{ m} \times 28\text{ m}\\ &= 182\text{ m}^2\end{aligned}$$

$$\begin{aligned}A &= bh\\ &= 22\text{ m} \times 28\text{ m}\\ &= 616\text{ m}^2\end{aligned}$$



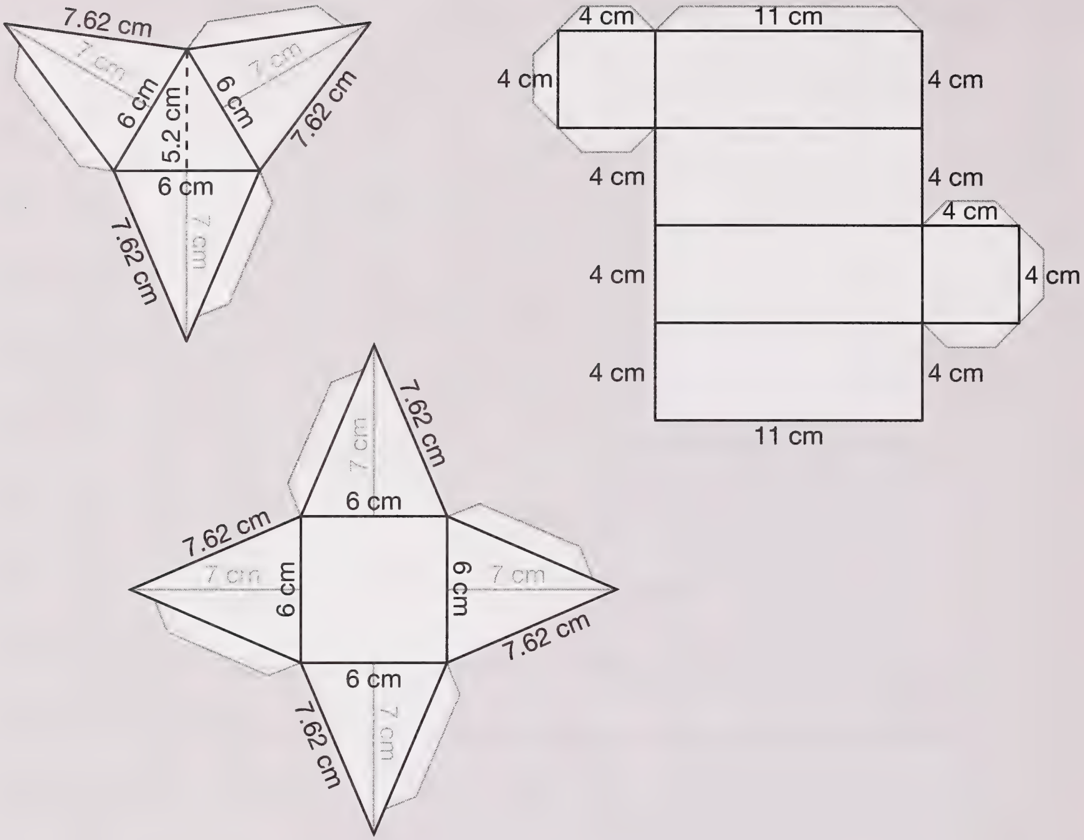
The lot has an area of $182\text{ m}^2 + 616\text{ m}^2 = 798\text{ m}^2$.

Lesson 2: Solving Area Problems with 3-D Solids

1. Textbook, page 164, "Investigation 1: How can you determine the surface area of 3-dimensional solids?", questions 1 to 7

1. You were asked to use the nets provided at the end of the Appendix.

2.



3. To calculate the surface area, find the area of each rectangle and triangle in the shape. Add the values together for the total surface area.

4. The surface area of the rectangular prism can be found by adding the areas of the two squares to the area of the four rectangles.

Square	**Rectangle**
$A = s^2$	$A = b \times h$
$= 4\text{ cm} \times 4\text{ cm}$	$= 11\text{ cm} \times 4\text{ cm}$
$= 16\text{ cm}^2$	$= 44\text{ cm}^2$

This gives a total surface area of 208 cm^2.

$$\left(16\text{ cm}^2 \times 2\right) + \left(4 \times 44\text{ cm}^2\right) \quad \text{two squares plus four rectangles}$$
$$= 32\text{ cm}^2 + 176\text{ cm}^2$$
$$= 208\text{ cm}^2$$

The surface area of the square pyramid can be found by adding the area of the central square to the areas of the four outer triangles.

Square

$$\begin{aligned} A &= s^2 \\ &= 6 \text{ cm} \times 6 \text{ cm} \\ &= 36 \text{ cm}^2 \end{aligned}$$

Triangle

$$\begin{aligned} A &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 6 \text{ cm} \times 7 \text{ cm} \\ &= 21 \text{ cm}^2 \end{aligned}$$

This gives a total surface area of 120 cm^2.

$$\begin{aligned} &36 \text{ cm}^2 + \left(4 \times 21 \text{ cm}^2\right) \quad \text{central square plus four triangles} \\ &= 36 \text{ cm}^2 + 84 \text{ cm}^2 \\ &= 120 \text{ cm}^2 \end{aligned}$$

The surface area of the triangular pyramid can be found by adding the area of the central triangle to the areas of the three outer triangles.

Triangle

$$\begin{aligned} A &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 6 \text{ cm} \times 5.2 \text{ cm} \\ &= 15.6 \text{ cm}^2 \end{aligned}$$

Triangle

$$\begin{aligned} A &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 6 \text{ cm} \times 7 \text{ cm} \\ &= 21 \text{ cm}^2 \end{aligned}$$

This gives a total surface area of 78.6 cm^2.

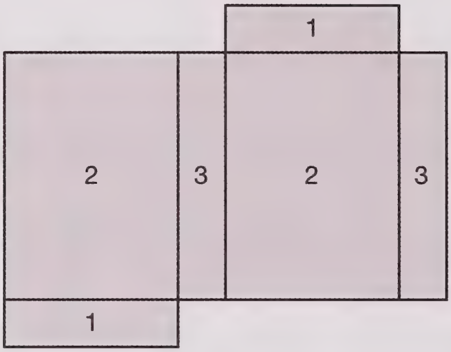
$$\begin{aligned} 15.6 \text{ cm}^2 + \left(3 \times 21 \text{ cm}^2\right) &= 15.6 \text{ cm}^2 + 63 \text{ cm}^2 \quad \text{central triangle plus three outer triangles} \\ &= 78.6 \text{ cm}^2 \end{aligned}$$

5. and **6.** You will get a rectangular prism and two pyramids.

7. To find the surface area of any 3-D object, create a net for the object. Then break the net into rectangles and other shapes that are easy to find areas for. Finally, add the areas of the pieces to get a total surface area.

2. **Textbook, page 165, "Investigation 2: Are there formulas for surface area?", questions 1 and 2**

1. **a.** The net for a cereal box would look similar to the one that follows.



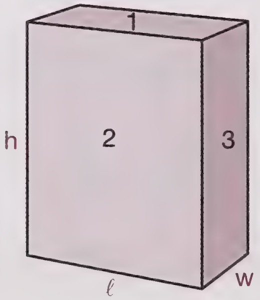
b. There are six faces.

c. Each face is a rectangle.

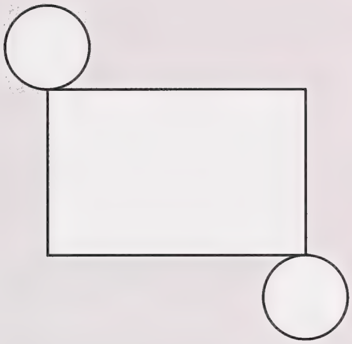
d. There are three pairs of faces that are the same size. They are marked with the same number in the net.

e. Measure the base and height of each rectangle. The area of the rectangle is the product of these measurements.

f. surface area $= 2 \times \left[(\ell \times w) + (\ell \times h) + (w \times h) \right]$



2. a. The net for a cylinder would look similar to the one at the right.



b. There are three surfaces.

c. There are two circles and a rectangle.

d. The two circles are the same size and shape.

e. The area of each face is calculated as follows. Sample numbers are used for the calculations.

Rectangle	**Circle**
$A = b \times h$	$A = \pi r^2$
$= 18.8 \times 11$	$= \pi (3)^2$
$= 206.8$	$\doteq 28.274\ 333\ 88$

f. surface area $= 2\pi\left(r^2 + r \times h\right)$

g. You would not need to add the two circles if the cylinder was open at both ends. The formula would become much simpler.

surface area $= 2\pi(r \times h)$

3. Textbook, pages 166 and 167, "Put into Practice," questions 1 to 4

1. a. i. The net is made up of six squares. To find the surface area of the net, you multiply the area of one square by 6.

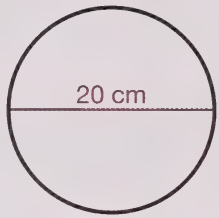
ii. The net is made up of two circles and a rectangle. To find the surface area of the net, multiply the area of the circle by 2 and add the area of the rectangle.

iii. The net is made up of four triangles. To find the surface area of the net, multiply the area of one triangle by 4.

b. i. This net would give a cube as the solid
ii. This net would give a cylinder as the solid.
iii. This net would give a triangular pyramid as the solid.

2. The gerbil wheel is like an open cylinder. The material needed can be thought of as a rectangle that is 6 cm wide and 20 π cm long. You can also think of it as the circumference of a circle that has a height of 6 cm.

$$\begin{aligned} A &= \text{base} \times \text{height} \\ &= (\pi \times d) \times h \\ &= \pi \times 20\text{ cm} \times 6\text{ cm} \\ &= 120\pi\text{ cm}^2 \\ &\doteq 376.991\ 118\ 4\text{ cm}^2 \end{aligned}$$



$$\begin{aligned} C &= \pi d \\ &= 20\pi \end{aligned}$$

The gerbil wheel will require 377 cm^2 of material.

3. Darlene will have to find the area of the square and add it to 4 times the area of one triangle.

Square

$$\begin{aligned} A &= s^2 \\ &= (2.5\text{ cm})^2 \\ &= 6.25\text{ cm}^2 \end{aligned}$$

Triangle

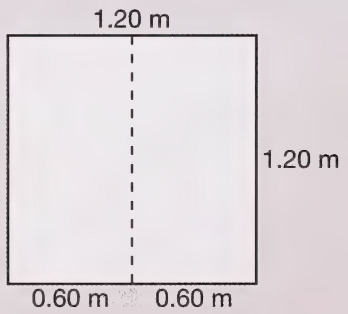
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= 0.5 \times 2.5\text{ cm} \times 3.0\text{ cm} \\ &= 3.75\text{ cm}^2 \end{aligned}$$

Four Triangles

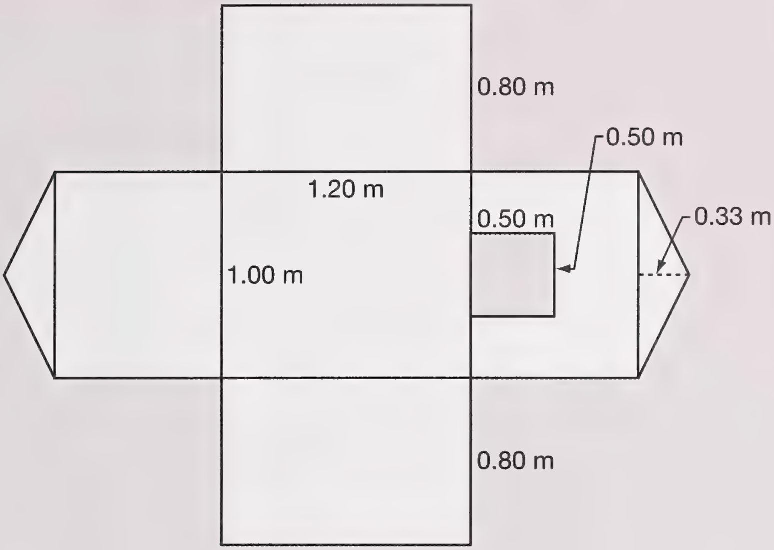
$$\begin{aligned} A &= 4 \times \text{area of one triangle} \\ &= 4 \times 3.75\text{ cm}^2 \\ &= 15\text{ cm}^2 \end{aligned}$$

The area of material is 6.25 cm^2 + 15 cm^2 = 21.25 cm^2. Rounded to the nearest square centimetre, this is 21 cm^2.

4. a. You can create the net for the roof several ways. A sample answer follows.



You can create the net for the walls and floor several ways. A sample answer follows.



b. The area of the roof is found as follows.

$$\begin{aligned} A &= bh \\ &= 1.20\text{ m} \times 1.20\text{ m} \\ &= 1.4400\text{ m}^2 \end{aligned}$$

The area of the floor and walls is calculated in five parts.

Floor

$$\begin{aligned} A &= bh \\ &= 1.20\text{ m} \times 1.00\text{ m} \\ &= 1.2000\text{ m}^2 \end{aligned}$$

Long Walls

$$\begin{aligned} A &= bh \\ &= 1.20\text{ m} \times 0.80\text{ m} \\ &= 0.9600\text{ m}^2 \end{aligned}$$

Short Walls

$$\begin{aligned} A &= bh \\ &= 0.80\text{ m} \times 1.00\text{ m} \\ &= 0.8000\text{ m}^2 \end{aligned}$$

Door

$$\begin{aligned} A &= bh \\ &= 0.50\text{ m} \times 0.50\text{ m} \\ &= 0.2500\text{ m}^2 \end{aligned}$$

Triangles

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= 0.5 \times 1.00\text{ m} \times 0.33\text{ m} \\ &= 0.165\text{ m}^2 \end{aligned}$$

The surface area is the sum of the areas of the floor, two long walls, two short walls, and two triangles, minus the area of the door.

$$A = 1.2000\text{ m}^2 + \left(2\times 0.9600\text{ m}^2\right) + \left(2\times 0.8000\text{ m}^2\right) + \left(2\times 0.165\text{ m}^2\right) - 0.2500\text{ m}^2$$
$$= 4.800\text{ m}^2$$

The outside surface area of the doghouse, rounded to the nearest square metre, is 5 m^2.

c. The inside surface area is the same as the outside surface area. The total inside and outside surface area would be 5 m^2 + 5 m^2 = 10 m^2. Kushtrim will need to buy 5 L of paint to paint the doghouse. $(10 \div 2 = 5)$

d. The cost of the paint is found by multiplying the cost per litre by the number of litres needed.

$$\text{cost} = 5 \times \$11.98$$
$$= \$59.90$$

The cost of paint would be $59.90.

Lesson 3: Are Perimeter and Area Related?

1. **Textbook, pages 168 and 169, "Investigation 1: What happens to area when you change the dimensions?", questions 1 to 3**

   1. **a.** to **e.** The answers are shown in the two spreadsheets that follow.

	A	B	C	D
1	**Perimeter**	**Length**	**Width**	**Area**
2	24	1	=(A2-2*B2)/2	=B2*C2
3	24	=B2+1	=(A3-2*B3)/2	=B3*C3
4	24	=B3+1	=(A4-2*B4)/2	=B4*C4
5	24	=B4+1	=(A5-2*B5)/2	=B5*C5
6	24	=B5+1	=(A6-2*B6)/2	=B6*C6
7	24	=B6+1	=(A7-2*B7)/2	=B7*C7
8	24	=B7+1	=(A8-2*B8)/2	=B8*C8
9	24	=B8+1	=(A9-2*B9)/2	=B9*C9
10	24	=B9+1	=(A10-2*B10)/2	=B10*C10
11	24	=B10+1	=(A11-2*B11)/2	=B11*C11
12	24	=B11+1	=(A12-2*B12)/2	=B12*C12

	A	B	C	D
1	**Perimeter**	**Length**	**Width**	**Area**
2	24.00	1.00	11.00	11.00
3	24.00	2.00	10.00	20.00
4	24.00	3.00	9.00	27.00
5	24.00	4.00	8.00	32.00
6	24.00	5.00	7.00	35.00
7	24.00	6.00	6.00	36.00
8	24.00	7.00	5.00	35.00
9	24.00	8.00	4.00	32.00
10	24.00	9.00	3.00	27.00
11	24.00	10.00	2.00	20.00
12	24.00	11.00	1.00	11.00

f. The dog run with the largest area has dimensions of 6 m by 6 m. The dog run with the smallest area has dimensions of 11 m by 1 m.

g. A larger area is probably better for the dog.

2. a. The following spreadsheets show the area calculations.

i.

	A	B	C	D
1	**Perimeter**	**Length**	**Width**	**Area**
2	18.00	1.00	8.00	8.00
3	18.00	2.00	7.00	14.00
4	18.00	3.00	6.00	18.00
5	18.00	4.00	5.00	20.00
6	18.00	5.00	4.00	20.00
7	18.00	6.00	3.00	18.00
8	18.00	7.00	2.00	14.00
9	18.00	8.00	1.00	8.00

ii.

	A	B	C	D
1	**Perimeter**	**Length**	**Width**	**Area**
2	16.00	1.00	7.00	7.00
3	16.00	2.00	6.00	12.00
4	16.00	3.00	5.00	15.00
5	16.00	4.00	4.00	16.00
6	16.00	5.00	3.00	15.00
7	16.00	6.00	2.00	12.00
8	16.00	7.00	1.00	7.00

iii.

	A	B	C	D
1	Perimeter	Length	Width	Area
2	20.00	1.00	9.00	9.00
3	20.00	2.00	8.00	16.00
4	20.00	3.00	7.00	21.00
5	20.00	4.00	6.00	24.00
6	20.00	5.00	5.00	25.00
7	20.00	6.00	4.00	24.00
8	20.00	7.00	3.00	21.00
9	20.00	8.00	2.00	16.00
10	20.00	9.00	1.00	9.00
11				

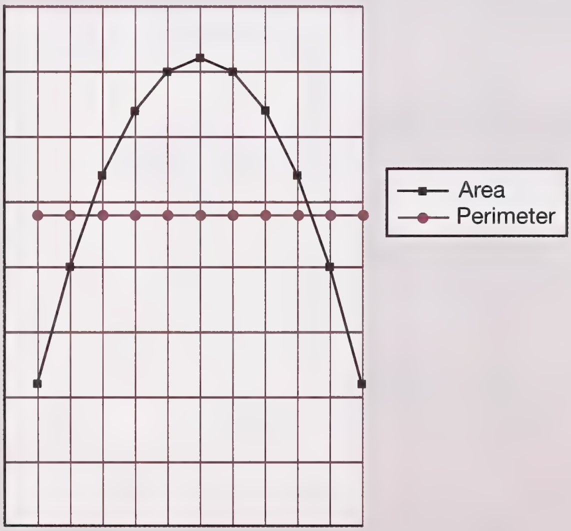
b. The largest areas come from these sizes of dog run.

i. 4 m by 5 m
ii. 4 m by 4 m
iii. 5 m by 5 m

The smallest areas come from these sizes of dog run.

i. 1 m by 8 m
ii. 1 m by 7 m
iii. 1 m by 9 m

3. If the perimeter doesn't change but the shape does change, the area changes. It seems to increase gradually to a largest value and then decrease back to the minimum. See the following graph. The closer the shape is to a square, the larger the area.



2. Textbook, pages 169 and 170, "Investigation 2: Changing Perimeter," questions 1 to 3

1. a. to **e.** The following two spreadsheets show the answers.

	A	B	C	D
1	**Area**	**Length**	**Width**	**Perimeter**
2	12	1	=A2/B2	=2*(B2+C2)
3	=A2	=B2+1	=A3/B3	=2*(B3+C3)
4	=A3	=B3+1	=A4/B4	=2*(B4+C4)
5	=A4	=B4+1	=A5/B5	=2*(B5+C5)
6	=A5	=B5+1	=A6/B6	=2*(B6+C6)
7	=A6	=B6+1	=A7/B7	=2*(B7+C7)
8	=A7	=B7+1	=A8/B8	=2*(B8+C8)
9	=A8	=B8+1	=A9/B9	=2*(B9+C9)
10	=A9	=B9+1	=A10/B10	=2*(B10+C10)
11	=A10	=B10+1	=A11/B11	=2*(B11+C11)
12	=A11	=B11+1	=A12/B12	=2*(B12+C12)
13	=A12	=B12+1	=A13/B13	=2*(B13+C13)
14				

	A	B	C	D
1	**Area**	**Length**	**Width**	**Perimeter**
2	12	1	12	26
3	12	2	6	16
4	12	3	4	14
5	12	4	3	14
6	12	5	2.4	14.8
7	12	6	2	16
8	12	7	1.714285714	17.42857143
9	12	8	1.5	19
10	12	9	1.333333333	20.66666667
11	12	10	1.2	22.4
12	12	11	1.090909091	24.18181818
13	12	12	1	26
14				

f. The largest perimeter comes from the 1-unit by 12-unit shape. The smallest perimeter comes from the 3-unit by 4-unit shape.

g. Answers may vary. It probably depends on the size of the lizard and its habits.

2. a. i.

	A	B	C	D
1	**Area**	**Length**	**Width**	**Perimeter**
2	14	1	14	30
3	14	2	7	18
4	14	3	4.6666667	15.333333
5	14	4	3.5	15
6	14	5	2.8	15.6
7	14	6	2.3333333	16.666667
8	14	7	2	18
9	14	8	1.75	19.5
10	14	9	1.5555556	21.111111
11	14	10	1.4	22.8
12	14	11	1.2727273	24.545455
13	14	12	1.1666667	26.333333
14	14	13	1.0769231	24.153846
15	14	14	1	30

ii.

	A	B	C	D
1	**Area**	**Length**	**Width**	**Perimeter**
2	24	1	24	50
3	24	2	12	28
4	24	3	8	22
5	24	4	6	20
6	24	5	4.8	19.6
7	24	6	4	20
8	24	7	3.4285714	20.857143
9	24	8	3	22
10	24	9	2.6666667	23.333333
11	24	10	2.4	24.8
12	24	11	2.1818182	26.363636
13	24	12	2	28
14	24	13	1.8461538	29.692308
15	24	14	1.7142857	31.428571
16	24	15	1.6	33.2
17	24	16	1.5	35
18	24	17	1.4117647	36.823529
19	24	18	1.3333333	38.666667
20	24	19	1.2631579	40.526316
21	24	20	1.2	42.4
22	24	21	1.1428571	44.285714
23	24	22	1.0909091	46.181818
24	24	23	1.0434783	48.086957
25	24	24	1	50

iii.

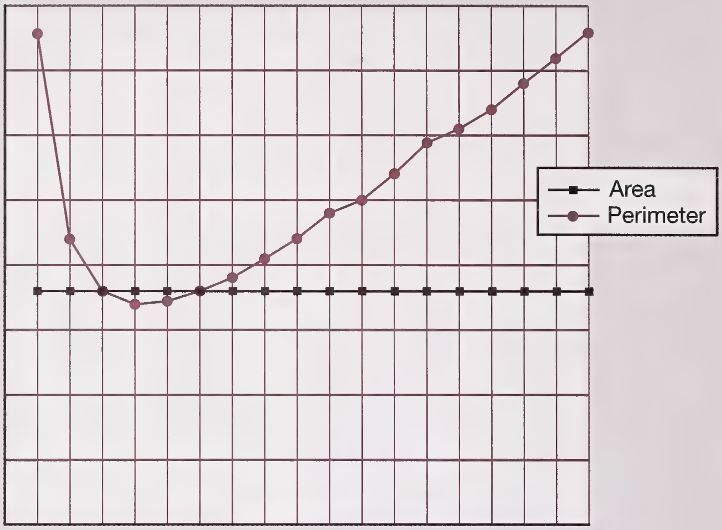
	A	B	C	D
1	Area	Length	Width	Perimeter
2	18	1	18	38
3	18	2	9	22
4	18	3	6	18
5	18	4	4.5	17
6	18	5	3.6	17.2
7	18	6	3	18
8	18	7	2.5714286	19.142857
9	18	8	2.25	20.5
10	18	9	2	22
11	18	10	1.8	23.6
12	18	11	1.6363636	25.272727
13	18	12	1.5	27
14	18	13	1.3846154	28.769231
15	18	14	1.2857143	30.571429
16	18	15	1.2	32.4
17	18	16	1.125	34.25
18	18	17	1.0588235	36.117647
19	18	18	1	38
20				

b. **i.** The largest perimeter comes from a 1-unit by 14-unit base. The smallest perimeter comes from a 2-unit by 7-unit base (if all dimensions are whole numbers).

ii. The largest perimeter comes from a 1-unit by 24-unit base. The smallest perimeter comes from a 4-unit by 6-unit base (if all dimensions are whole numbers).

iii. The largest perimeter comes from a 1-unit by 18-unit base. The smallest perimeter comes from a 3-unit by 6-unit base (if all dimensions are whole numbers).

3. If the area doesn't change but the shape does change, the perimeter changes. It seems to decrease rapidly to a smallest value and then increase slowly back to the maximum. See the graph to the right. The closer to a square the shape becomes, the smaller the perimeter is.

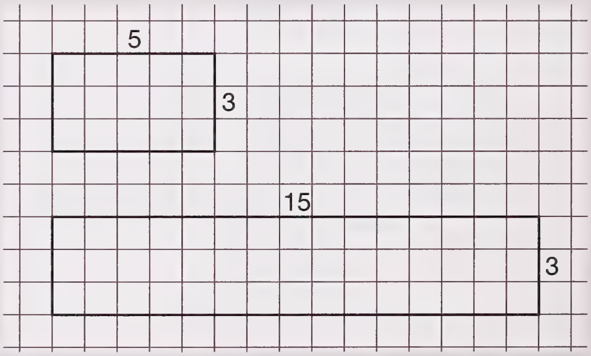


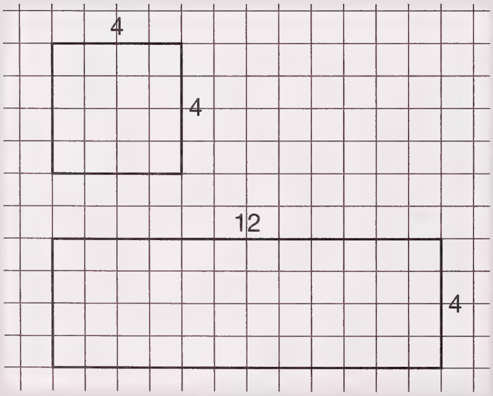
3. **Textbook, page 170, "Investigation 3: Perimeter and Area Patterns," questions 1 to 4**

1. The following diagrams and calculations show that the area increases by a factor of 3. They also show that the perimeter increases by 4 times the length. (Because the questions talk about length and width, the formula $A = \ell \times w$ is used to find the area.)

$$A = \ell \times w = 5 \times 3 = 15 \qquad A = \ell \times w = 15 \times 3 = 45$$

$$P = 2(\ell + w) = 2(5+3) = 16 \qquad P = 2(\ell + w) = 2(15+3) = 36$$





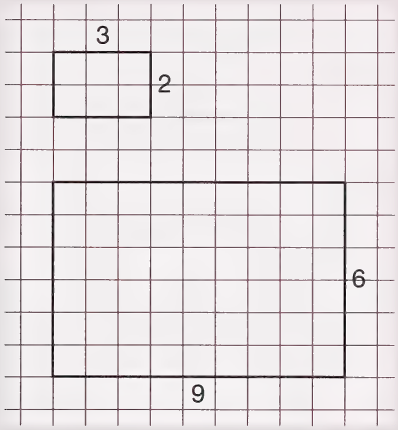
$$A = \ell \times w = 4 \times 4 = 16 \qquad A = \ell \times w = 12 \times 4 = 48$$

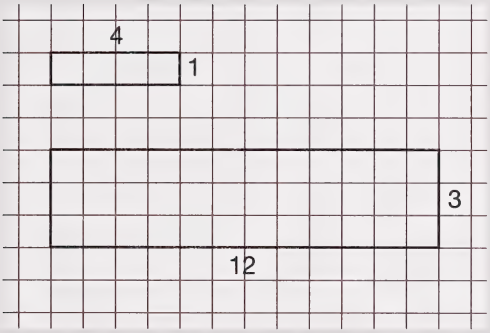
$$P = 2(\ell + w) = 2(4+4) = 16 \qquad P = 2(\ell + w) = 2(12+4) = 32$$

2. The following diagrams and calculations show that the area increases by a factor of 9. They also show that the perimeter increases by a factor of 3.

$$\begin{aligned} A &= \ell \times w \\ &= 3 \times 2 \\ &= 6 \end{aligned} \qquad \begin{aligned} A &= \ell \times w \\ &= 9 \times 6 \\ &= 54 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(3+2) \\ &= 10 \end{aligned} \qquad \begin{aligned} P &= 2(\ell + w) \\ &= 2(9+6) \\ &= 30 \end{aligned}$$





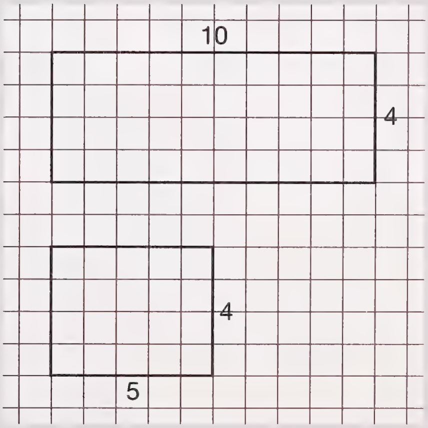
$$\begin{aligned} A &= \ell \times w \\ &= 4 \times 1 \\ &= 4 \end{aligned} \qquad \begin{aligned} A &= \ell \times w \\ &= 12 \times 3 \\ &= 36 \end{aligned}$$

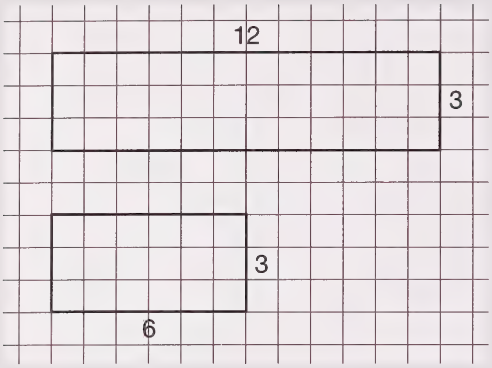
$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(4+1) \\ &= 10 \end{aligned} \qquad \begin{aligned} P &= 2(\ell + w) \\ &= 2(12+3) \\ &= 30 \end{aligned}$$

3. The following diagrams and calculations show that the area is halved. They also show that the perimeter decreases by the length.

$$\begin{aligned} A &= \ell \times w \\ &= 10 \times 4 \\ &= 40 \end{aligned} \qquad \begin{aligned} A &= \ell \times w \\ &= 5 \times 4 \\ &= 20 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(10+4) \\ &= 28 \end{aligned} \qquad \begin{aligned} P &= 2(\ell + w) \\ &= 2(5+4) \\ &= 18 \end{aligned}$$





$$\begin{aligned} A &= \ell \times w \\ &= 12 \times 3 \\ &= 36 \end{aligned}$$

$$\begin{aligned} A &= \ell \times w \\ &= 6 \times 3 \\ &= 18 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(12 + 3) \\ &= 30 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(6 + 3) \\ &= 18 \end{aligned}$$

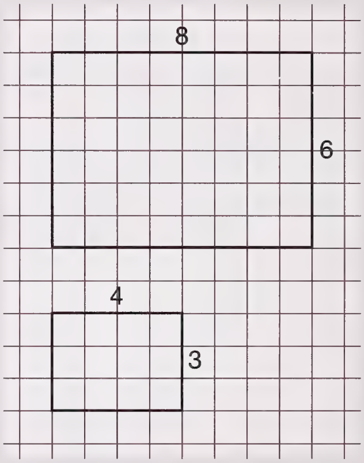
4. The following diagrams and calculations show that the area is divided by 4. They also show that the perimeter is divided by 2.

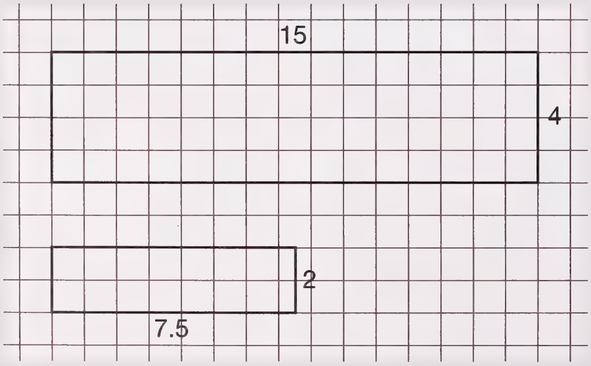
$$\begin{aligned} A &= \ell \times w \\ &= 8 \times 6 \\ &= 48 \end{aligned}$$

$$\begin{aligned} A &= \ell \times w \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(8 + 6) \\ &= 28 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(4 + 3) \\ &= 14 \end{aligned}$$





$$\begin{aligned} A &= \ell \times w \\ &= 15 \times 4 \\ &= 60 \end{aligned}$$

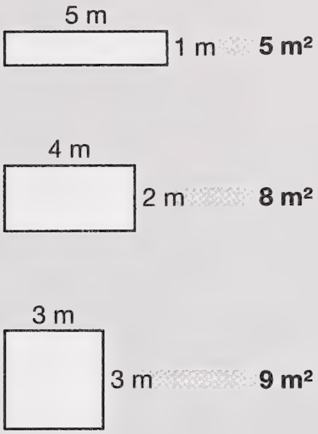
$$\begin{aligned} A &= \ell \times w \\ &= 7.5 \times 2 \\ &= 15 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(15 + 4) \\ &= 38 \end{aligned}$$

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(7.5 + 2) \\ &= 19 \end{aligned}$$

4. **Textbook, page 171, "Put into Practice," questions 1 to 3**

1. The largest area comes from a square. The largest area that can be enclosed is 9 m^2.



2. **a.** She should make the pen a square with sides measuring 2.5 m.

   b. The pen would have an area of 6.25 m^2.

$$\begin{aligned} A &= s^2 \\ &= (2.5\text{ m})^2 \\ &= 6.25\text{ m}^2 \end{aligned}$$

   c. **i.** The diameter of a circle with a circumference of 10 m is found as follows:

$$\begin{aligned} C &= \pi d \\ 10\text{ m} &= \pi d \\ \frac{10\text{ m}}{\pi} &= \frac{\pi d}{\pi} \\ d &= \frac{10\text{ m}}{\pi} \\ d &\doteq 3.183\ 098\ 862\text{ m} \end{aligned}$$

The diameter, rounded to the nearest tenth of a metre, is 3.2 m.

ii. The area of the circular pen would be found as follows:

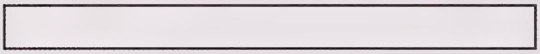
$$
\begin{aligned}
A &= \pi r^2 \\
&= \pi \times \left(\frac{3.2\ \text{m}}{2}\right)^2 \\
&= 2.56\pi\ \text{m}^2 \\
&\doteq 8.042\ 477\ 193\ \text{m}^2
\end{aligned}
$$

The area of the circular pen, correct to one decimal place, would be 8.0 m^2.

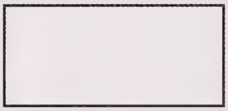
iii. Suzette should make a circular pen because it provides the greatest area.

3. a. She should make a square exercise yard with sides of 70 m.

7 m by 700 m uses 1414 m.



20 m by 245 m uses 530 m.



49 m by 100 m uses 298 m.

70 m by 70 m uses 280 m.

b. The radius of a circular pen with an area of 4900 m^2 is calculated as follows:

$$
\begin{aligned}
A &= \pi r^2 \\
4900\ \text{m}^2 &= \pi r^2 \\
r^2 &= \frac{4900\ \text{m}^2}{\pi} \\
r &= \sqrt{\frac{4900}{\pi}}\ \text{m} \\
r &\doteq 39.493\ 270\ 85\ \text{m}
\end{aligned}
$$

The circumference of a circle with a radius of 39.493 270 85 m is found as follows:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 39.493\ 270\ 85 \text{ m} \\ &\doteq 248.143\ 539\ 1 \text{ m} \end{aligned}$$

A circular exercise yard with an area of 4900 m^2 would require about 248 m of fencing.

Section 3: Measurement and Volume

Lesson 1: Enlargements and Reductions

1. Textbook, pages 172 to 174, "Investigation: How do they compare?", questions 1 to 6

1. **a.** The small squares in B are 15 mm tall.

   b. The small squares in A are 5 mm tall.

   c. The ratio of the heights of the squares in B and A is $\frac{15}{5} = \frac{3}{1}$.

2. **a.** The area of a small square in B is 225 mm^2.

$$\begin{aligned} A &= s^2 \\ &= (15 \text{ mm})^2 \\ &= 225 \text{ mm}^2 \end{aligned}$$

   b. The area of a small square in A is 25 mm^2.

$$\begin{aligned} A &= s^2 \\ &= (5 \text{ mm})^2 \\ &= 25 \text{ mm}^2 \end{aligned}$$

   c. The ratio of the areas of squares in B and A is $\frac{225 \text{ mm}^2}{25 \text{ mm}^2} = \frac{9}{1}$.

3. **a.** The ratio of the length of a square in B to the length of a square in A is $\frac{15 \text{ mm}}{5 \text{ mm}} = \frac{3}{1}$.

   b. The ratio of the area of a square in B to the area of a square in A is $\frac{225 \text{ mm}^2}{25 \text{ mm}^2} = \frac{9}{1}$.

   c. The ratio of areas is the square of the ratio of lengths.

$$\begin{aligned} \frac{9}{1} &= \frac{3^2}{1^2} \\ &= \left(\frac{3}{1}\right)^2 \end{aligned}$$

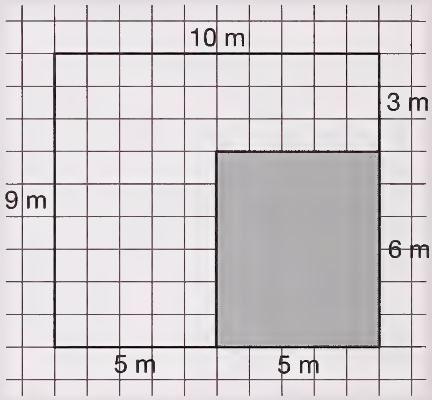
d. Was your pattern the same as the one described in the answer to question 3.c.?

e. If you double all the dimensions, the area will increase by a factor of 4. For example, if a 1-cm square was doubled in length and width to give a square that's 2 cm on each side, the area would increase from 1 cm^2 to 4 cm^2.

f. **i.** The area of the 3-cm by 5-cm rectangle is 15 cm^2. When its length and width are doubled, giving a 6-cm by 10-cm rectangle, the area becomes 60 cm^2.

$$\begin{aligned} A &= b \times h \\ &= 5\text{ cm} \times 3\text{ cm} \\ &= 15\text{ cm}^2 \end{aligned} \qquad \begin{aligned} A &= b \times h \\ &= (5\text{ cm} \times 2) \times (3\text{ cm} \times 2) \\ &= 10\text{ cm} \times 6\text{ cm} \\ &= 60\text{ cm}^2 \end{aligned}$$

ii. The area of this shape can be found as the difference of a large rectangle (9 m by 10 m) and a smaller rectangle (5 m by 6 m).

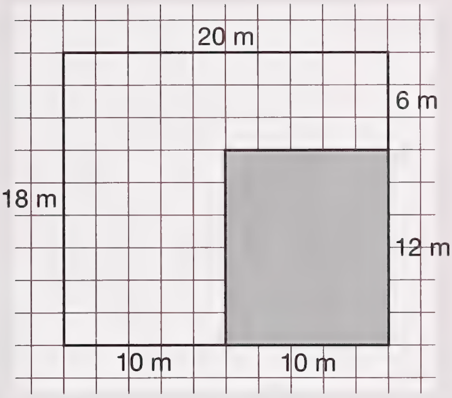


$$\begin{aligned} A &= b \times h \\ &= 10\text{ m} \times 9\text{ m} \\ &= 90\text{ m}^2 \end{aligned} \qquad \begin{aligned} A &= b \times h \\ &= 5\text{ m} \times 6\text{ m} \\ &= 30\text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= A_L - A_S \\ &= 90\text{ m}^2 - 30\text{ m}^2 \\ &= 60\text{ m}^2 \end{aligned}$$

The area of the given shape is 60 m^2.

The area of this shape can be found as the difference of a large rectangle (18 m by 20 m) and a smaller rectangle (10 m by 12 m).



$$\begin{aligned} A &= b \times h \\ &= 20 \text{ m} \times 18 \text{ m} \\ &= 360 \text{ m}^2 \end{aligned} \qquad \begin{aligned} A &= b \times h \\ &= 10 \text{ m} \times 12 \text{ m} \\ &= 120 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= A_L - A_S \\ &= 360 \text{ m}^2 - 120 \text{ m}^2 \\ &= 240 \text{ m}^2 \end{aligned}$$

When the dimensions are all doubled, the area becomes 240 m^2.

4. **a.** The perimeter of a shape will double if you double its dimensions.

b. **i.** The perimeter of this shape is 16 cm. When the dimensions are doubled, the perimeter becomes 32 cm.

$$\begin{aligned} P &= 2(\ell + w) \\ &= 2(5 \text{ cm} + 3 \text{ cm}) \\ &= 16 \text{ cm} \end{aligned} \qquad \begin{aligned} P &= 2(\ell + w) \\ &= 2[(5 \text{ cm} \times 2) + (3 \text{ cm} \times 2)] \\ &= 2(10 \text{ cm} + 6 \text{ cm}) \\ &= 32 \text{ cm} \end{aligned}$$

ii. The perimeter of this shape is 38 m. When the dimensions are doubled, the perimeter becomes 76 m.

$$\begin{aligned} P &= 10 \text{ m} + 9 \text{ m} + 5 \text{ m} + 6 \text{ m} + 5 \text{ m} + 3 \text{ m} \\ &= 38 \text{ m} \end{aligned}$$

$$\begin{aligned} P &= 20 \text{ m} + 18 \text{ m} + 10 \text{ m} + 12 \text{ m} + 10 \text{ m} + 6 \text{ m} \\ &= 76 \text{ m} \end{aligned}$$

5. **a.** The ratio of the length of the toe of the enlarged picture to that of the original picture is $\frac{3}{1}$. The ratio of the length of a toe is the same as the ratio of the length of a side of the squares drawn over the frog.

b. The ratio of the area of the enlarged frog to that of the original frog is $\frac{9}{1}$. This is the same as the ratio of the area of an enlarged square to that of the original square.

6. **a.** The ratio of the lengths of turtles C and D is $\frac{5}{2}$.

   b. The ratio of the areas of turtles C and D is $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$.

   c. The area ratio is the square of the ratio of the sides. Area can be thought of as the product of length and width, so if each changes in the same way, the area will change in the square of that way.

2. Textbook, page 174, "Put into Practice," questions 1 to 5

1. This can be solved using a proportion. Let x be the real distance from Banff to Calgary.

$$\frac{1}{1\,600\,000} = \frac{6.5 \text{ cm}}{x}$$
$$x = 6.5 \text{ cm} \times 1\,600\,000$$
$$x = 10\,400\,000 \text{ cm}$$

$$\begin{aligned}10\,400\,000 \text{ cm} &= 10\,400\,000 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}\\ &= 104\,000 \text{ m}\\ &= 104\,000 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}}\\ &= 104 \text{ km}\end{aligned}$$

The distance is about 10 400 000 cm or 104 000 m or 104 km.

2. The length of the enlarged image is calculated as follows:

$$\begin{aligned}\text{enlarged length} &= 40 \times \text{actual length}\\ &= 40 \times 0.3 \text{ mm}\\ &= 12.0 \text{ mm}\end{aligned}$$

The enlarged length is 12 mm.

3. The orca in the picture has a length of 2.25 inches. The scale is calculated as follows:

$$\begin{aligned}\text{scale} &= \frac{\text{drawing}}{\text{real}}\\ &= \frac{2.25 \text{ in}}{22 \text{ ft}}\\ &= \frac{2.25 \text{ in}}{22 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}}}\\ &= \frac{2.25 \text{ in}}{264 \text{ in}}\\ &\doteq \frac{1}{117.333\,333\,3}\end{aligned}$$

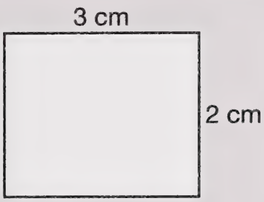
The scale is about 1:117.

4. The dimensions of the drawing are as follows:

   - Thirty feet at a scale of 1 in = 4 ft becomes 7.5 in. $(30 \div 4 = 7.5)$
   - Twenty-three feet at a scale of 1 in = 4 ft becomes 5.75 in. $(23 \div 4 = 5.75)$

   You need a rectangle with the above dimensions.

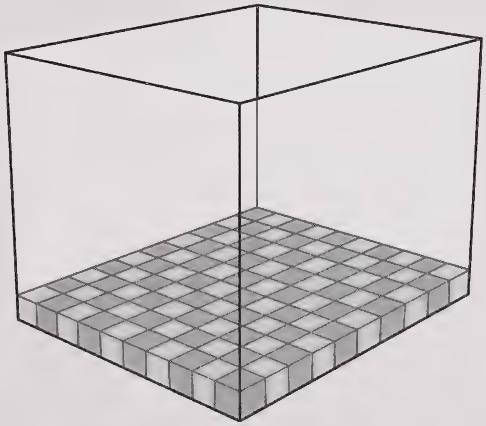
5. The scale for this drawing is 1:100.



Lesson 2: Solving Volume Problems

1. **Textbook, pages 176 and 177, "Investigation 1: Can you develop a formula for determining volume?", questions 1 to 3**

   1. **a.** to **c.** It will take 100 1-cm cubes to fill the bottom layer.



   d. It will take 10 layers to fill the cube.

   e. To fill the cube, there will be 10 layers of 100 cubes, for a total of 1000 cubes.

   f. The volume of the cube is 1000 cm^3.

   2. **a.** It would take 25 cubes to fill the bottom layer. It would take 10 layers of 25 cubes each, or 250 cubes in all, to fill the prism.

   b. The pattern is to multiply the length, width, and height together. This product is the volume.

   c. Did you arrive at the same pattern?

   d. $V = A \times h$

3. a. This formula works for solids that don't change shape as they change height.
   b. If the cross section of the solid changes, this formula will not work.

2. Textbook, pages 177 to 179, "Investigation 2: Volumes of Other Prisms," questions 1 to 4

1. **a.** and **b.** It would take 3 layers of 4 green triangles. This is 12 triangles in all.
   c. It would take 10 layers of 4 triangles, or 40 triangles in all.

2. **a.** This triangle has an area of 60 cm^2.

   b. It would take a minimum of 60 such cubes.

   c. If the prism was 5 cm high, it would take 5 layers of 60 cubes, or 300 cubes in all.

   d. If the prism was 20 cm high, it would take 20 layers of 60 cubes, or 1200 cubes in total.

   e. The pattern is to multiply the number of cubes to cover the base by the number of layers of cubes needed to fill the prism.

   f. Did you arrive at the same pattern?

   g. $V = A \times h$

   h. This is the same as the formula from Investigation 1.

3. **a.** The area is about 314 cm^2.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (10\text{ cm})^2 \\ &\doteq 314.159\,265\,4\text{ cm}^2 \end{aligned}$$

   b. It would take at least 314 cubes to fill the prism.

   c. It would take 314 more cubes to fill the prism if the height increased by 1 cm.

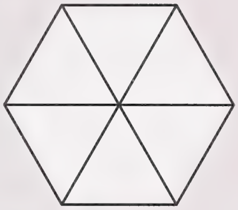
d. The pattern is to multiply the area of the base by the height.

e. Is this the same pattern you found?

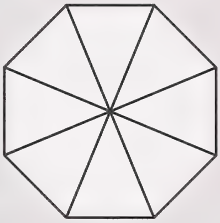
f. $V = A \times h$

g. This is the same formula created in Investigation 1 and Investigation 2.

4. a. The base is a hexagon. It can be broken into 6 triangles. Use the formula for the area of a triangle to find the area of each triangle making up the base of the hexagon. Add the areas of these triangles together and multiply this area by the height of the prism. This will give the volume of the prism.



b. The base is an octagon. It can be broken into 8 triangles. Use the formula for the area of a triangle to find the area of each triangle making up the base of the hexagon. Add the areas of these triangles together and multiply this area by the height of the prism. This will give the volume of the prism.



c. The base is a triangle. Use the formula for the area of a triangle to find the area of the base of this prism. Multiply this area by the height of the prism to find the volume.

3. Textbook, page 181 and 182, "Put into Practice," questions 1 to 6

You might have used the formula $V = A \times h$ and continued with $V = (b \times h) \times h$. This can cause confusion because of the two h symbols. These solutions all use $V = A \times h = (\ell \times w) \times h$ to prevent this confusion.

1. a.

Estimate	**Calculate**
$V = A \times h$	$V = A \times h$
$\doteq (\ell \times w) \times h$	$= (\ell \times w) \times h$
$\doteq 1 \text{ in} \times 5 \text{ in} \times 5 \text{ in}$	$= 2 \text{ in} \times 4 \text{ in} \times 6 \text{ in}$
$\doteq 25 \text{ in}^3$	$= 48 \text{ in}^3$

The volume is 48 in^3.

b. **Estimate**

$$V = A \times h$$
$$= (\ell \times w) \times h$$
$$\doteq 40\text{ cm} \times 30\text{ cm} \times 70\text{ cm}$$
$$\doteq 84\ 000\text{ cm}^3$$

Calculate

$$V = A \times h$$
$$= (\ell \times w) \times h$$
$$= 43\text{ cm} \times 28\text{ cm} \times 68\text{ cm}$$
$$= 81\ 872\text{ cm}^3$$

The volume is 81 872 cm^3.

c. **Estimate**

$$V = A \times h$$
$$= (\ell \times w) \times h$$
$$\doteq 70\text{ m} \times 30\text{ m} \times 40\text{ m}$$
$$\doteq 84\ 000\text{ m}^3$$

Calculate

$$V = A \times h$$
$$= (\ell \times w) \times h$$
$$= 72.8\text{ m} \times 26.2\text{ m} \times 35.9\text{ m}$$
$$= 68\ 474.224\text{ m}^3$$

The volume is 68 474.224 m^3.

2. a. **Estimate**

$$V = A \times h$$
$$= (\pi r^2)h$$
$$\doteq 3 \times (4\text{ cm})^2 \times 10\text{ cm}$$
$$\doteq 480\text{ cm}^3$$

Calculate

$$V = A \times h$$
$$= (\pi r^2)h$$
$$= \pi \times (4\text{ cm})^2 \times 10\text{ cm}$$
$$= 502.654\ 824\ 6\text{ cm}^3$$

The volume is about 502.7 cm^3.

b. **Estimate**

$$V = A \times h$$
$$= (\pi r^2)h$$
$$\doteq 3 \times (10\text{ in})^2 \times 5\text{ in}$$
$$\doteq 1500\text{ in}^3$$

Calculate

$$V = A \times h$$
$$= (\pi r^2)h$$
$$= \pi \times (9\text{ in})^2 \times 7\text{ in}$$
$$\doteq 1781.283\ 035\text{ in}^3$$

The volume is about 1781.3 in^3.

3. a. The concrete will be 0.12 m thick since $12 \times 0.01 = 0.12$.

$$\begin{aligned} V &= A \times h \\ &= (\ell \times w) \times h \\ &= 5\text{ m} \times 4\text{ m} \times 0.12\text{ m} \\ &= 2.4\text{ m}^3 \end{aligned}$$

The floor will take 2.4 m^3 of concrete.

b. The cost of the floor is calculated as follows:

$$\begin{aligned} \text{cost} &= 2.4\text{ m}^3 \times \$30/\text{m}^3 \\ &= \$72.00 \end{aligned}$$

The floor will cost $72.00.

4. a. The small bale has a volume of 9 ft^3.

$$\begin{aligned} V &= A \times h \\ &= (\ell \times w) \times h \\ &= 3\text{ ft} \times 1.5\text{ ft} \times 2\text{ ft} \\ &= 9.0\text{ ft}^3 \end{aligned}$$

b. The large bale has a volume of 128 ft^3.

$$\begin{aligned} V &= A \times h \\ &= (\ell \times w) \times h \\ &= 4\text{ ft} \times 4\text{ ft} \times 8\text{ ft} \\ &= 128\text{ ft}^3 \end{aligned}$$

c. The large cylindrical bale has a volume of about 98 ft^3.

$$\begin{aligned} V &= A \times h \\ &= (\pi r^2)h \\ &= \pi \times \left(\frac{5\text{ ft}}{2}\right)^2 \times 5\text{ ft} \\ &\doteq 98.174\ 770\ 42\text{ ft}^3 \end{aligned}$$

d. The costs of the other bales can be calculated as follows. Let x be the cost of the type of bale being found.

Large Square Bale

$$\frac{x}{128\text{ ft}^3} = \frac{\$10}{9\text{ ft}^3}$$

$$\frac{\cancel{128\text{ ft}^3} \times x}{\cancel{128\text{ ft}^3}} = \frac{128\ \cancel{\text{ft}^3} \times \$10}{9\ \cancel{\text{ft}^3}}$$

$$x = \frac{128 \times \$10}{9}$$

$$x \doteq \$142.222\ 222\ 2$$

Large Cylindrical Bale

$$\frac{x}{98.174\ 770\ 42\text{ ft}^3} = \frac{\$10}{9\text{ ft}^3}$$

$$\frac{\cancel{98.174\ 770\ 42\text{ ft}^3} \times x}{\cancel{98.174\ 770\ 42\text{ ft}^3}} = \frac{98.174\ 770\ 42\ \cancel{\text{ft}^3} \times \$10}{9\ \cancel{\text{ft}^3}}$$

$$x = \frac{98.174\ 770\ 42 \times \$10}{9}$$

$$x \doteq \$109.083\ 078\ 2$$

5. The volume can be calculated as follows:

$$\begin{aligned} V &= A \times h \\ &= \left(\pi r^2\right)h \\ &= \pi \times \left(\frac{2.5\text{ in}}{2}\right)^2 \times 15\text{ in} \\ &\doteq 73.631\ 077\ 82\text{ in}^3 \end{aligned}$$

The bird feeder can hold 73.6 in^3 of birdseed.

6. The height can be calculated as follows:

$$\begin{aligned} V &= A \times h \\ 68\ 400\text{ cm}^3 &= (\ell \times w) \times h \\ 68\ 400\text{ cm}^3 &= (30\text{ cm} \times 60\text{ cm}) \times h \\ \frac{68\ 400\text{ cm}^3}{30\text{ cm} \times 60\text{ cm}} &= \frac{30\text{ cm} \times 60\text{ cm} \times h}{30\text{ cm} \times 60\text{ cm}} \\ 38\text{ cm} &= h \end{aligned}$$

The height needs to be 38 cm to give the snake enough room.

Lesson 3: Other Types of Measurement

1. **Textbook, page 183, "Investigation: SI Units," questions i. to vii.**

   i. 100 metres or 100 m
   ii. 90 kilometres per hour or 90 km/h
   iii. 1.2 metres per second or 1.2 m/s
   iv. 35 kilometres per hour or 35 km/h
   v. 6 tonnes or 6 t
   vi. 225 litres or 225 L
   vii. 6 metres or 6 m

2. **Textbook, page 184 to 187, "Put into Practice," questions 1, 2, 3, 5, 7, and 9**

   1. The elephant eats 1925 kg of food each week. This gives an average of 275 kg each day $(1925 \div 7 = 275)$. The fraction of its body mass that it eats each day is $\frac{1}{20}$.

   $$\frac{275\text{ kg}}{5500\text{ kg}} = \frac{\cancel{5 \times 5 \times 11}}{2 \times 2 \times 5 \times \cancel{5 \times 5 \times 11}} = \frac{1}{20}$$

   2. The amount of food eaten by an Orca whale in a week is 1120 kg $(160 \times 7 = 1120)$. Since 1120 kg is 25% of its body weight, the Orca whale's body weight is 4480 kg.

   $$1120\text{ kg} = 25\%\text{ of } x$$
   $$1120\text{ kg} = 0.25x$$
   $$\frac{\overset{4480}{\cancel{1120}}\text{ kg}}{\underset{1}{\cancel{0.25}}} = \frac{\overset{1}{\cancel{0.25}}x}{\underset{1}{\cancel{0.25}}}$$
   $$4480\text{ kg} = x$$

3. This can be solved as follows:

Price per Kilogram in Dollars

$$\frac{x}{1\text{ kg}} = \frac{\$6.59}{1.8\text{ kg}}$$

$$\frac{x \times \cancel{1\text{ kg}}}{\cancel{1\text{ kg}}} = \frac{\$6.59 \times \cancel{1\text{ kg}}}{1.8\,\cancel{\text{kg}}}$$

$$x = \frac{\$6.59}{1.8}$$

$$x \doteq \$3.661111111$$

Price per Gram in Cents

$$\frac{x}{1\text{ g}} = \frac{659¢}{1800\text{ g}}$$

$$\frac{x \times \cancel{1\text{ g}}}{\cancel{1\text{ g}}} = \frac{659¢ \times 1\,\cancel{\text{g}}}{1800\,\cancel{\text{g}}}$$

$$x = \frac{659¢}{1800}$$

$$x \doteq 0.366\,1111111¢$$

The cost per kilogram of the dog food is about \$3.66. (\$3.66/kg)

The cost per gram of the dog food is about 0.366¢. (\$0.366¢/g)

5. There are 3600 seconds in an hour. There are 1000 m in 1 km.

$$100\text{ km/h} = \frac{\overset{100}{\cancel{100\text{ km}}}}{\cancel{\text{h}}} \times \frac{1\,\cancel{\text{h}}}{\underset{36}{\cancel{3600}}\text{ s}} \times \frac{\overset{10}{\cancel{1000}}\text{ m}}{\underset{1}{\cancel{1\text{ km}}}}$$

$$= \frac{1000\text{ m}}{36\text{ s}}$$

$$\doteq 27.777\,777\,78\text{ m/s}$$

The cheetah can travel 27.8 m in one second.

7. There are 3600 seconds in an hour. There are 1000 m in 1 km.

$$\frac{200\text{ m}}{10\text{ s}} = \frac{\overset{1}{\cancel{200\text{ m}}}}{\underset{1}{\cancel{10\text{ s}}}} \times \frac{\overset{360}{\cancel{3600\text{ s}}}}{1\text{ h}} \times \frac{1\text{ km}}{\underset{5}{\cancel{1000\text{ m}}}}$$

$$= \frac{360\text{ km}}{5\text{ h}}$$

$$= 72\text{ km/h}$$

The ostrich is traveling at 72 km/h.

9. **a.** This can be solved by finding the volume of water.

$$\begin{aligned} V &= A \times h \\ &= (\ell \times w) \times h \\ &= (150\text{ cm} \times 60\text{ cm}) \times 25\text{ cm} \\ &= 9000\text{ cm}^2 \times 25\text{ cm} \\ &= 225\,000\text{ cm}^3 \end{aligned}$$

There are 225 000 cm^3 of water in the tank. This is 225 000 mL of water.

b. A litre of water is made up of 1000 mL. There are 225 L of water in the tank. $(225\,000 \div 1000 = 225)$

c. A kilogram is the same as 1000 g. Each millilitre of water is the same as 1 g. There are 225 kg of water in the tank. $(225\,000 \div 1000 = 225)$

d. There are many possible reasons for needing this information. Two examples are given:

- to see if a stand is strong enough to hold the filled tank
- to see if it would be easy to move the filled tank

Review

Textbook, pages 188 to 191, "Review of Unit Three," questions 1 to 8

1. **a.** To find the perimeter of a rectangle, use the formula $P = 2\ell + 2w$.

To find the area of a rectangle, use one of the formulas $A = b \times h$ or $A = \ell \times w$.

$$\begin{aligned} P &= 2\ell + 2w \\ &= (2 \times 12\text{ cm}) + (2 \times 6.5\text{ cm}) \\ &= 24\text{ cm} + 13\text{ cm} \\ &= 37\text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \ell \times w \\ &= 12\text{ cm} \times 6.5\text{ cm} \\ &= 78\text{ cm}^2 \end{aligned}$$

The perimeter of the given rectangle is 37 cm. The area of the given rectangle is 78 cm^2.

b. To find the perimeter of a square, use one of the formulas $P = 2\ell + 2w$ or $P = 4s$.

To find the area of a square, use one of the formulas $A = b \times h$, $A = \ell \times w$, or $A = s^2$.

$$\begin{aligned} P &= 4s \\ &= 4 \times 3.8 \text{ in} \\ &= 15.2 \text{ in} \end{aligned} \qquad \begin{aligned} A &= s^2 \\ &= (3.8 \text{ in})^2 \\ &= 14.44 \text{ in}^2 \end{aligned}$$

The perimeter of the square is 15.2 inches. The area of the square is about 14.4 in^2.

c. To find the perimeter of a triangle, use the formula $P = s_1 + s_2 + s_3$.

To find the area of a triangle, use the formula $A = \frac{1}{2} b \times h$.

$$\begin{aligned} P &= s_1 + s_2 + s_3 \\ &= 5 \text{ ft} + 5 \text{ ft} + 6 \text{ ft} \\ &= 16 \text{ ft} \end{aligned} \qquad \begin{aligned} A &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 6 \text{ ft} \times 4 \text{ ft} \\ &= 12 \text{ ft}^2 \end{aligned}$$

The perimeter of the given triangle is 16 ft. The area of the given triangle is 12 ft^2.

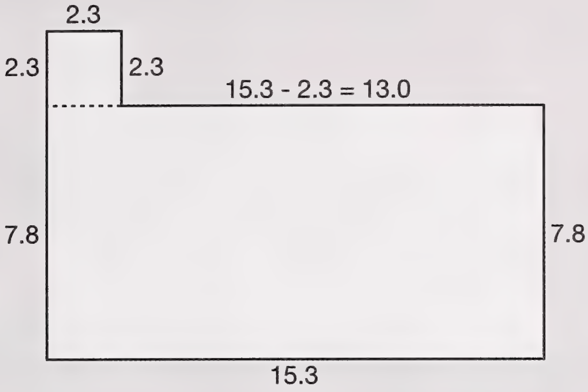
d. The perimeter of a circle is also known as its circumference. The circumference of a circle is found using the formula $C = \pi d$ or $C = 2\pi r$.

The area of a circle is found using the formula $A = \pi r^2$.

$$\begin{aligned} P &= \pi d \\ &= \pi \times 10 \text{ km} \\ &\doteq 31.415\,926\,54 \text{ km} \end{aligned} \qquad \begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{10 \text{ km}}{2} \right)^2 \\ &\doteq 78.539\,816\,34 \text{ km}^2 \end{aligned}$$

The perimeter (circumference) of the given circle is about 31.4 km. The area of the given circle is about 78.5 km^2.

e. This is a more complex shape made up of a small square and a large rectangle.



The small square has sides measuring 2.3 cm. The rectangle has a length of 15.3 cm and a width of 7.8 cm.

$$\begin{aligned} P &= s_1 + s_2 + s_3 + s_4 + s_5 + s_6 \\ &= 10.1\ \text{cm} + 2.3\ \text{cm} + 2.3\ \text{cm} + 13.0\ \text{cm} + 7.8\ \text{cm} + 15.3\ \text{cm} \\ &= 50.8\ \text{cm} \end{aligned}$$

$$\begin{aligned} A &= s^2 \\ &= (2.3\ \text{cm})^2 \\ &= 5.29\ \text{cm}^2 \end{aligned}$$

$$\begin{aligned} A &= \ell \times w \\ &= 15.3\ \text{cm} \times 7.8\ \text{cm} \\ &= 119.34\ \text{cm}^2 \end{aligned}$$

$5.29\ \text{cm}^2 + 119.34\ \text{cm}^2 = 124.63\ \text{cm}^2$

The perimeter of the shape is 50.8 cm. The area of the shape is 124.6 cm^2.

f. This is a more complex shape made up of a triangle and a rectangle. The triangle has a base of 30 m and a height of 40 m (240 m – 200 m). The rectangle has a length of 30 m and a width of 200 m.

$$\begin{aligned} P &= s_1 + s_2 + s_3 + s_4 \\ &= 30\ \text{m} + 200\ \text{m} + 50\ \text{m} + 240\ \text{m} \\ &= 520\ \text{m} \end{aligned}$$

$$\begin{aligned} A &= b \times h \\ &= 30\ \text{m} \times 200\ \text{m} \\ &= 6000\ \text{m}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 30\ \text{m} \times 40\ \text{m} \\ &= 600\ \text{m}^2 \end{aligned}$$

$6000\ \text{m}^2 + 600\ \text{m}^2 = 6600\ \text{m}^2$

The perimeter of the shape is 520 m. The area of the shape is 6600 m^2.

2. a. The shaded area is the area of the circle minus the area of the triangle.

Area of Circle

$$A = \pi r^2$$
$$= \pi\left(\frac{10 \text{ cm}}{2}\right)^2$$
$$= 25\pi \text{ cm}^2$$
$$\doteq 78.539\,816\,34 \text{ cm}^2$$

Area of Triangle

$$A = \frac{1}{2} b \times h$$
$$= \frac{1}{2} \times 8.5 \text{ cm} \times 7.5 \text{ cm}$$
$$= 31.875 \text{ cm}^2$$

$78.539\,816\,34 \text{ cm}^2 - 31.875 \text{ cm}^2 = 46.664\,816\,34 \text{ cm}^2$

The shaded area is 46.7 cm^2.

b. The shaded area is the area of the large rectangle minus the area of the small square and the small parallelogram.

Area of Rectangle

$$A = \ell \times w$$
$$= 18 \text{ cm} \times 12 \text{ cm}$$
$$= 216 \text{ cm}^2$$

Area of Square

$$A = s^2$$
$$= (4 \text{ cm})^2$$
$$= 16 \text{ cm}^2$$

Area of Parallelogram

$$A = b \times h$$
$$= 9 \text{ cm} \times 2 \text{ cm}$$
$$= 18 \text{ cm}^2$$

Since $216 \text{ cm}^2 - 16 \text{ cm}^2 - 18 \text{ cm}^2 = 182 \text{ cm}^2$, the shaded area is 182 cm^2.

c. The shaded area is the area of the parallelogram minus the area of the rectangle and the semicircle.

Area of Parallelogram

$$A = b \times h$$
$$= 18 \text{ cm} \times 10 \text{ cm}$$
$$= 180 \text{ cm}^2$$

Area of Rectangle

$$A = \ell \times w$$
$$= 3 \text{ cm} \times 8 \text{ cm}$$
$$= 24 \text{ cm}^2$$

Area of Semicircle

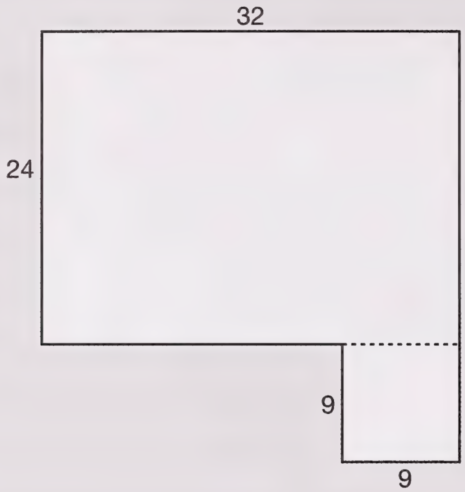
$$A = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi (2 \text{ cm})^2$$
$$= 6.283\,185\,307 \text{ cm}^2$$

Since $180 \text{ cm}^2 - 24 \text{ cm}^2 - 6.283\,185\,307 \text{ cm}^2 = 149.716\,814\,7 \text{ cm}^2$, the shaded area is about 149.7 cm^2.

3. a. The possible shapes follow.

b. Answers will vary. Most likely, a long and narrow enclosure would provide the best place for a dog to run.

4. The question describes a shape that is made up of two rectangles. The situation is shown in the following diagram.

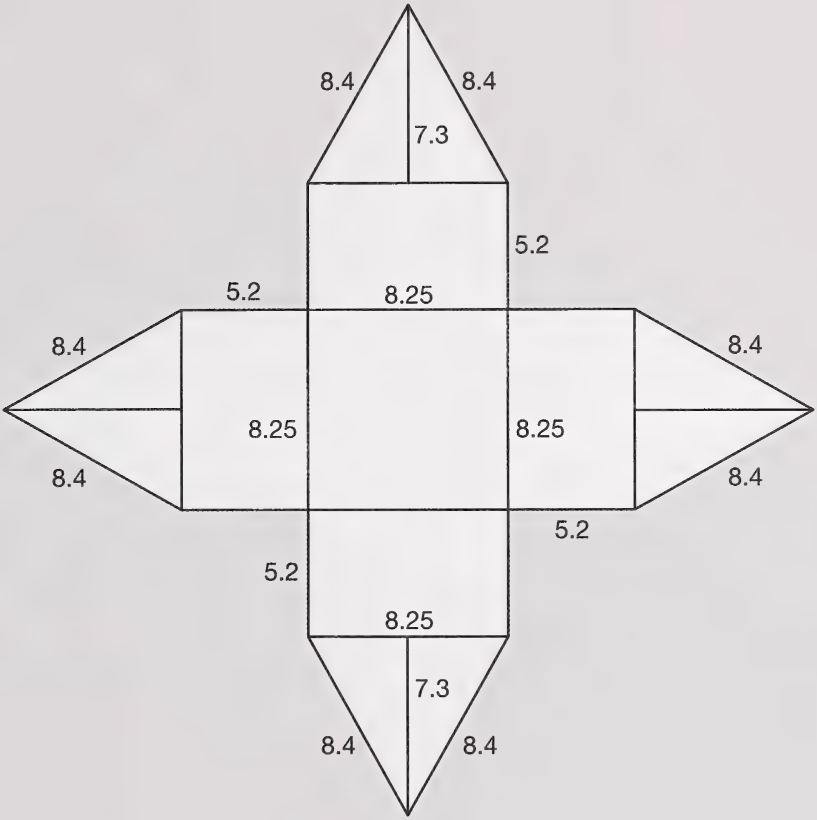


The area of the shape is the sum of the areas of the two rectangles.

Area of Large Rectangle	**Area of Square**	**Total Area**
$A = \ell \times w$ $= 32 \text{ ft} \times 24 \text{ ft}$ $= 768 \text{ ft}^2$	$A = s^2$ $= (9 \text{ ft})^2$ $= 81 \text{ ft}^2$	$768 \text{ ft}^2 + 81 \text{ ft}^2 = 849 \text{ ft}^2$

The calf has an area of 849 ft^2 set aside for it.

5. **a.** To fit the net on a standard size page, a scale of 1 cm:2 dm was used.



b. The glass area is made up of four triangles and four rectangles.

Area of One Rectangle

$$A = \ell \times w$$
$$= 8.25 \text{ dm} \times 5.2 \text{ dm}$$
$$= 42.9 \text{ dm}^2$$

Area of One Triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 8.25 \text{ dm} \times 7.3 \text{ dm}$$
$$= 30.1125 \text{ dm}^2$$

Area of Four Rectangles

$$4 \times 42.9 \text{ dm}^2 = 171.6 \text{ dm}^2$$

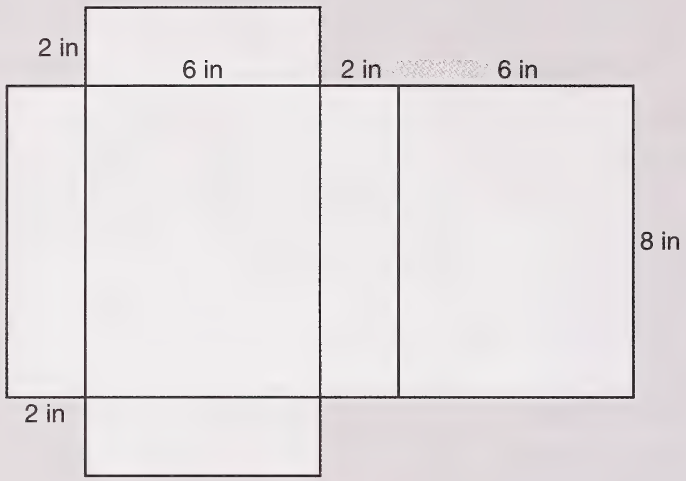
Area of Four Triangles

$$4 \times 30.1125 \text{ dm}^2 = 120.45 \text{ dm}^2$$

Since $171.6 \text{ dm}^2 + 120.45 \text{ dm}^2 = 292.05 \text{ dm}^2$, the area, to the nearest square decimetre, is 292 dm^2.

c. Each edge of the base is 8.25 dm. There are four edges to connect to the base. The total length of the seams is 4×8.25 dm $= 33$ dm.

6. A sketch of the net for the box follows.



The net shows the box is made of three sizes of rectangles.

Area of Top (and Bottom)

$$\begin{aligned} A &= b \times h \\ &= 6 \text{ in} \times 8 \text{ in} \\ &= 48 \text{ in}^2 \end{aligned}$$

Area of Front (and Back)

$$\begin{aligned} A &= b \times h \\ &= 8 \text{ in} \times 2 \text{ in} \\ &= 16 \text{ in}^2 \end{aligned}$$

Area of Left (and Right)

$$\begin{aligned} A &= b \times h \\ &= 6 \text{ in} \times 2 \text{ in} \\ &= 12 \text{ in}^2 \end{aligned}$$

The total area is found as follows:

area of top + area of bottom = twice the area of the top
area of front + area of back = twice the area of the front
area of left + area of right = twice the area of the left

$$\begin{aligned} A &= (2 \times \text{top}) + (2 \times \text{front}) + (2 \times \text{left}) \\ &= \left(2 \times 48 \text{ in}^2\right) + \left(2 \times 16 \text{ in}^2\right) + \left(2 \times 12 \text{ in}^2\right) \\ &= 96 \text{ in}^2 + 32 \text{ in}^2 + 24 \text{ in}^2 \\ &= 152 \text{ in}^2 \end{aligned}$$

The total surface area of the box is 152 in^2.

7. **a.** The ratio of the enlarged length and the original length is 24:8 or, in reduced form, 3:1.

b. The ratio of the enlarged area to the original area will be 9:1.

Area of Enlargement

$$\begin{aligned} A &= b \times h \\ &= (8\text{ cm} \times 3) \times (6\text{ cm} \times 3) \\ &= 48\text{ cm}^2 \times 9 \\ &= 432\text{ cm}^2 \end{aligned}$$

Area of Original

$$\begin{aligned} A &= b \times h \\ &= 8\text{ cm} \times 6\text{ cm} \\ &= 48\text{ cm}^2 \end{aligned}$$

c. A 7.3-cm long fish in the original would be $7.3\text{ cm} \times 3 = 21.9\text{ cm}$ long in the enlargement.

8. First, convert the 50 km/h to an equivalent number of metres per second. Second, divide 100 m by the speed in metres per second to find the number of seconds.

$$\frac{50\text{ km}}{1\text{ h}} = \frac{50\text{ km}}{\cancel{1\text{ h}}} \times \frac{\cancel{1\text{ h}}}{60\text{ min}} \quad \text{First, change hours to minutes}$$

$$= \frac{50\text{ km}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}} \quad \text{Second, change minutes to seconds.}$$

$$= \frac{50\ \cancel{\text{km}}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \quad \text{Third, change kilometres to metres.}$$

$$= \frac{50\,000\text{ m}}{3600\text{ s}} \quad \text{Finally, simplify.}$$

$$\doteq 13.888\,888\,89\text{ m/s}$$

Since $100\text{ m} \div 13.888\,888\,89\text{ m/s} \doteq 7.2\text{ s}$, a grizzly could cover 100 m in 7.2 seconds.

Image Credits

All images in this courseware were created by or for Alberta Learning unless noted below.

Cover Photos: *left to right* EyeWire Collection/Getty Images, Will Woods/Digital Vision/Getty Images, PhotoDisc Collection/Getty Images
Title Page: PhotoDisc Collection/Getty Images
Welcome Page: PhotoDisc Collection/Getty Images
Contents Page: EyeWire Collection/Getty Images

Page

8 **both:** Corel Corporation
9 PhotoDisc Collection/Getty Images
10 **bottom:** RubberBall Productions/Getty Images
12 **both:** PhotoDisc Collection/Getty Images
14 **top and bottom:** RubberBall Productions/Getty Images
15 **top:** PhotoDisc Collection/Getty Images
16 Digital Vision/Getty Images
17 **bottom:** RubberBall Productions/Getty Images
18 **both:** PhotoDisc Collection/Getty Images
19 **both:** Corel Corporation
20 Digital Vision/Getty Images
21 **top:** RubberBall Productions/Getty Images
22 **top:** RubberBall Productions/Getty Images
bottom: PhotoDisc Collection/Getty Images
24 **top:** RubberBall Productions/Getty Images
25 **top:** RubberBall Productions/Getty Images
bottom: PhotoDisc Collection/Getty Images
26 **top:** RubberBall Productions/Getty Images
bottom: PhotoDisc Collection/Getty Images
29 RubberBall Productions/Getty Images
30 PhotoDisc Collection/Getty Images
31 **both:** PhotoDisc Collection/Getty Images
33 Brand X Pictures/Getty Images
34 **frog:** © 2003–2004 www.clipart.com
35 **top and middle:** © 2003–2004 www.clipart.com
bottom: PhotoDisc Collection/Getty Images
36 **top:** RubberBall Productions/Getty Images
bottom: PhotoDisc Collection/Getty Images
38 **top:** RubberBall Productions/Getty Images
39 Digital Vision/Getty Images
42 **both:** PhotoDisc Collection/Getty Images
43 PhotoDisc Collection/Getty Images

Appendix Title Page: PhotoDisc Collection/Getty Images

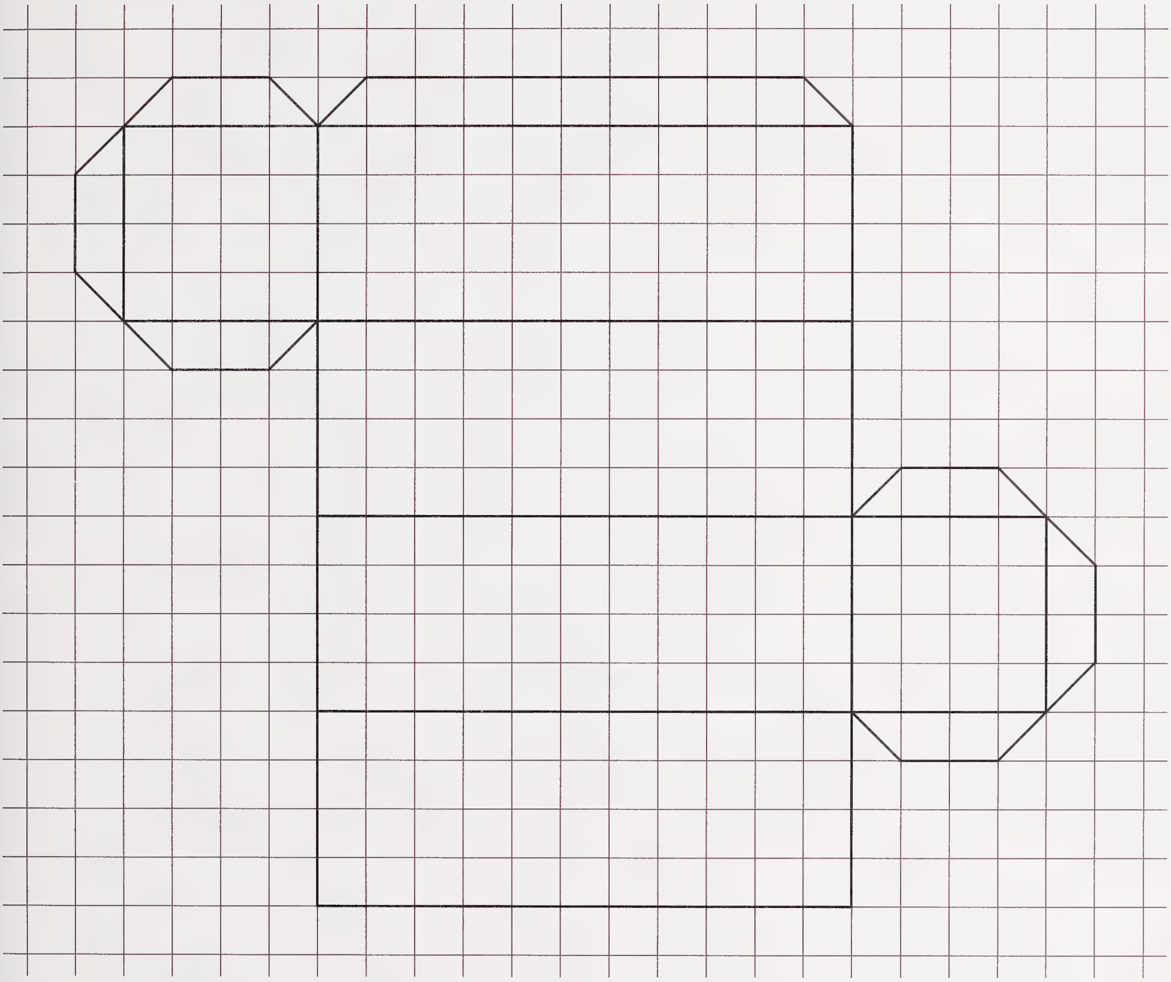
Spreadsheets and Multimedia

Following is a list of the spreadsheet templates and multimedia segments for Module 3 that appear on the multimedia CD:

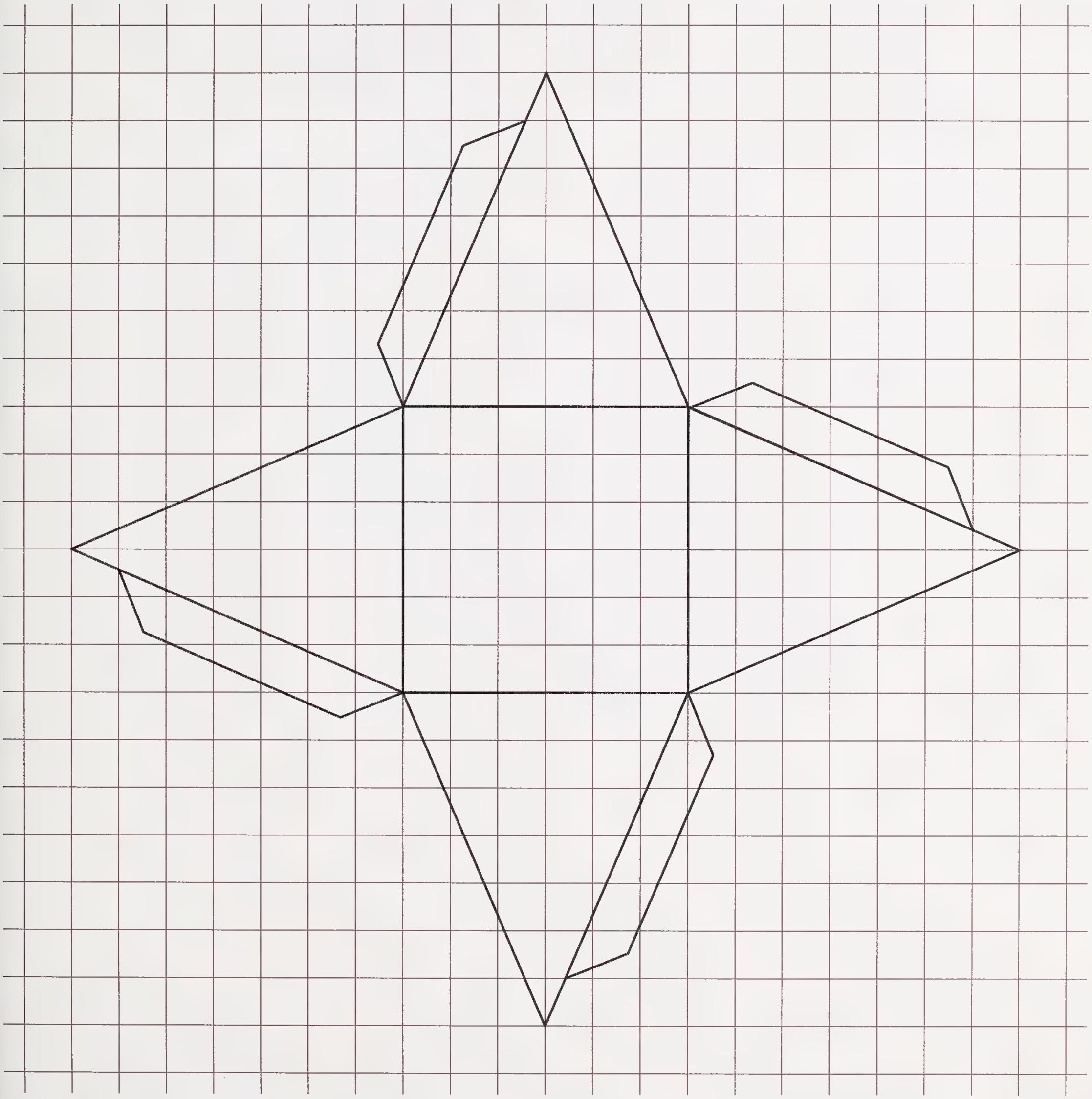
- The multimedia segment "Making Complicated Area Problems Simpler" lets you practise breaking complicated shapes into simpler ones.
- Mod_3_1.xlt finding the areas of different rectangles with a given perimeter
- Mod_3_2.xlt finding the perimeter of different rectangles with a given area

Learning Aids

Nets



Nets Continued



Nets Continued